165.

Problem 21.11 (RHK)

To circle the Earth in low orbit a satellite must have a speed of $7.91\ \text{km s}^{-1}$. *Suppose that two such satellites orbit the Earth in opposite directions. We have to find* (a) *their relative speed as they pass using the classical Galilean transformation equation;* (b) *the fractional error that is made because of not using the correct relativistic transformation equation.*

Solution:

(a)

According to classical Galilean transformation equations as the two satellites are moving with equal speed, *v*, $7.91\ \text{km s}^{-1}$ in opposite directions, their relative speed will be the arithmetic sum $\left(7.91+7.91\right)\ \text{km s}^{-1}=15.82\ \text{km s}^{-1}$.

(b)

According to the relativistic velocity addition their relative speed is given by the equation

$$v_{rel} = \frac{v+v}{1+v^2/c^2} = \frac{15.82\times10^3 \text{ m s}^{-1}}{1+\left(7.91\times10^3/3\times10^8\right)^2},$$

$$= \frac{15.82\times10^3 \text{ m s}^{-1}}{1+6.9\times10^{-10}},$$

$$\approx 15.82\times10^3\left(1-6.9\times10^{-10}\right) \text{ m s}^{-1},$$

$$= 15.82\times10^3 \text{ m s}^{-1} - 1.1\times10^{-5} \text{ m s}^{-1}.$$

Therefore, the fractional error in using the Galilean velocity addition is

$$E = \frac{1.1\times10^{-5}}{15.82\times10^3} = 6.95\times10^{-10}.$$