162.

Problem 20.71 (RHK)

A police car sounding its siren is moving at $27\ \mathrm{m\ s^{-1}}$ *and approaching a stationary pedestrian. The police in the car hear the siren at* $12.6\ \mathrm{kHz}$ *but the pedestrian hears the siren at* $13.7\ \mathrm{kHz}$. *We have to find the air temperature. (We can assume that the speed of sound increases linearly with temperature between* 0^0 *and* $20^0\ \mathrm{C}$*).*

$(v_s\left(T=0^0\ \mathrm{C}\right)=331\ \mathrm{m\ s^{-1}}$ and $v_s\left(T=20^0\ \mathrm{C}\right)=343\ \mathrm{m\ s^{-1}})$

Solution:

From the data we will calculate the speed of sound in air and from the speed we will find the temperature of air.

Speed of the police car, $v_{police}=27\ \mathrm{m\ s^{-1}}$.

Frequency of the police siren, $f=12.6\times10^3\ \mathrm{Hz}$.

Frequency of the siren as heard by the stationary pedestrian, $f_{ped}=13.7\times10^3\ \mathrm{Hz}$.

Let the speed of sound be v_s.

Relation between the frequencies is given by the Doppler shift relation

$$f_{ped} = f \frac{v_s}{v_s - v_{police}} ,$$

or

$$13.7\times10^3 = 12.6\times10^3 \frac{v_s}{v_s - 27 \text{ m s}^{-1}}.$$

From this equation, we get

$$v_s = 336.27 \text{ m s}^{-1}.$$

We will next calculate the coefficient of linear variation of speed of sound with temperature, α. We will use the data

$$v_s\left(T = 0^0 \text{ C}\right) = 331 \text{ m s}^{-1}$$

and

$$v_s\left(T = 20^0 \text{ C}\right) = 343 \text{ m s}^{-1}.$$

Let us assume linear variation of v_s with temperature.

$$v_s(T) = v_s\left(0^0\right) + \alpha T,$$

and

$$343=331+\alpha\times 20.$$

This gives

$$\alpha = \frac{12}{20} \text{ m s}^{-1} \text{ per } {}^{0}\text{C}.$$

Speed of sound of 336 m s^{-1} will be at the air temperature *T*,

$$T = \frac{5.27 \times 20}{12} = 8.78^{0}\text{C}.$$