159.

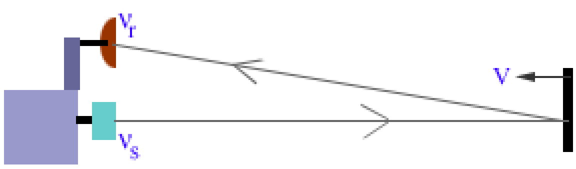
<u>Problem 20.65 (RHK)</u>

A device that contains a transmitter and a receiver of waves in a single instrument is used to measure the speed V of a target object (*idealised as a flat plate*) *that is moving directly toward the unit, by analysing waves reflected from it.* (a) *We have to apply the Doppler equations twice, first with the target as observer and with the target as a source, and have to show that the frequency* ν_r *of the reflected waves at the receiver is related to their source frequency* ν_s *by*

$$\nu_r = \nu_s\left(\frac{v+V}{v-V}\right),$$

where v is the speed of the waves. (b) *In great many practical applications, V = v. in this case, we have to show that the equation above becomes*

$$\frac{\nu_r - \nu_s}{\nu_s} \approx \frac{2V}{v}.$$



Solution:

(a)

Let the speed of the target which is moving toward the instrument be V.

Let the source frequency be ν_s and λ be the wavelength of the waves emitted by the transmitter of the instrument. Frequency of the waves received by the target, ν', by the Doppler effect will be

$$\nu' = \nu_s + \frac{V}{\lambda} = \nu_s + \frac{V}{v/\nu_s} = \nu_s\left(1 + \frac{V}{v}\right).$$

Now the moving target becomes a moving source. We will now calculate the frequency ν_r measured by the stationary detector.

Let λ_r be the wavelength measured by the detector. As the source is moving toward the detector with speed V and is emitting waves of frequency ν' the wavelength λ_r will be

$$\lambda_r = \frac{v}{\nu'} - \frac{V}{\nu'} = \frac{1}{\nu'}(v - V).$$

Substituting for ν', we get

$$\lambda_r = \frac{(v - V)}{\nu_s \left(1 + V/v\right)},$$

and

$$\nu_r = \frac{v}{\lambda_r} = \nu_s \frac{(v + V)}{(v - V)}.$$

(b)

In great many practical situations $V = v$, then

$$\frac{v + V}{v - V} \approx 1 + \frac{2V}{v}.$$

We thus find

$$\frac{\nu_r}{\nu_s} = 1 + \frac{2V}{v},$$

or

$$\frac{\nu_{\mathrm{r}} - \nu_s}{\nu_s} = \frac{2V}{v}.$$