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Problem 20.48 (RHK)

A 30.0-cm *violin string with linear mass density* $0.652\ \text{g m}^{-1}$ *is placed near a loudspeaker that is fed by an audio oscillator of variable frequency. It is found that the string is set into oscillation only at the frequencies* 880 *and* 1320 Hz *as the frequency of the oscillator is varied continuously over the range* 500-1500 Hz. *We have to find the tension in the string.*

Solution:

Length of the violin string, $L = 0.3$ m.

Linear mass density of the string, $\mu = 0.652\times10^{-3}\ \text{kg m}^{-1}$.

It is given that the string is set into oscillation the frequencies 880 Hz and 1320 Hz, when an oscillator is varied continuously over the frequency range 500-1500 Hz. Let the tension in the string be *F*. In a violin ends of its strings are clamped. So we are considering stationary states in a string with both ends being nodes. For stationary states the wavelength λ and the length are related as

$$\frac{\lambda}{2} n = L \text{ , where } n = 1, 2, 3, \ldots$$

Frequency at resonance can be found from the speed of sound in the string given by

$$v_s = \sqrt{\frac{F}{\mu}} \text{ ,}$$

and

$$f = \frac{v_s}{\lambda} = \frac{n v_s}{2L} \text{ .}$$

Let us assume that the two resonant frequencies correspond to $n = n_1$ and $n = n_1 + 1$. We then have an algebraic equation from which can solve for n_1. It is

$$\frac{f_2}{f_1} = \frac{1320}{880} = \frac{n_1 + 1}{n_1} \text{ ,}$$

or

$$440 n_1 = 880,$$

or

$$n_1 = 2 \text{ .}$$

We use the expression of f given in terms of F, L and n. It is

$$f = \frac{n}{2L} \times \sqrt{\frac{F}{\mu}}\ ,$$

or

$$F = \frac{4L^2 \mu f^2}{n_1^{\ 2}} = \frac{4 \times 0.3^2 \times 0.652 \times 10^{-3} \times 880^2}{4}\ \mathrm{N} = 45.4\ \mathrm{N}.$$