153.

Problem 20.41 (RHK)

A tunnel leading straight through a hill greatly amplifies tones at 135 *and* 138 Hz. *We have to find the shortest length the tunnel could have.*

Solution:

Tunnel can be regarded as a column with open ends. For pressure waves the ends of the tunnel, as they are open to atmosphere, at resonance will be nodes of standing waves. This requirement fixes wavelengths, λ, of standing waves at resonance in terms of the length of the tunnel, *L*, which we are considering an as open-ended resonance column. We have

$$\frac{\lambda}{2} \times n = L, \text{ where } n = 1, 2, 3, \ldots$$

We next calculate the wavelengths corresponding to frequencies $f = 135$ Hz and 138 Hz.

For $f = 135 \text{ Hz}$,

$$\lambda = \frac{343}{135} \text{ m} = 2.5407 \text{ m}.$$

For $f = 138$ Hz,

$$\lambda = \frac{343}{138} \text{ m} = 2.4855 \text{ m.}$$

We have to find integers n_1 and n_2 which will satisfy that

$$\frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1} \ ,$$

or

$$\frac{2.5407}{2.4855} = \frac{n_2}{n_1}.$$

As we are considering two successive overtones we may use that $n_2 = n_1 + 1$.

This implies that

$$2.5407 n_1 = \left(n_1 + 1\right) 2.485,$$

or

$$n_1 = \frac{2.485}{0.0552} = 45.0 \ .$$

From this result we can obtain the length of the tunnel. It is given by the relation that

$$L = \frac{n_1 \lambda}{2} = \frac{45 \times 2.5407}{2} \text{ m} = 57 \text{ m.}$$