152.

Problem 20.42 (RHK)

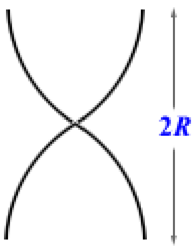
The period of a pulsating star may be estimated by considering the star to be executing radial longitudinal pulsations in the fundamental standing wave mode; that is, the radius varies periodically with time, with a displacement antinode at the surface. (a) *We have to answer whether the centre of the star will be a displacement node or antinode.* (b) *By making an analogy with the open organ pipe, we have to show that the period of pulsation is given by*

$$T = \frac{4R}{v_s} \ ,$$

where R is the equilibrium radius v_s *is the average speed of sound.* (c) *Typical white dwarf stars are composed of material with a bulk modulus of* 1.33×10^{22} Pa *and a density of* 1.0×10^{10} kg m^{-3}. *They have radius of the order of* 0.009 *solar radius. We have to estimate the approximate pulsation period of a white dwarf.*

Solution:

The period of a pulsating star may be estimated by modelling it to be a closed pipe executing longitudinal pulsations in the fundamental standing wave mode. The centre of the star has to be a pressure node as the pressure there is very large. In the fundamental standing wave mode the pulsations will look like as shown in the diagram.



Wavelength λ and the radius R are related as

$\lambda/2 = 2R,$ or $\lambda = 4R$.

Let v_s be the speed of longitudinal pressure oscillations of the material of the star. The speed v_s of the sound waves can be estimated from the elastic properties of the star. If B is the bulk modulus and ρ is the density of the star, the average speed of sound waves is given by

$$v_s = \sqrt{\frac{B}{\rho}} \ .$$

Frequency of oscillation in the fundamental mode will be

$$f = \frac{v_s}{\lambda} = \frac{v_s}{4R},$$

as

$$f = \frac{1}{T} \ ,$$

the period T of pulsations of the star will be

$$T = \frac{4R}{v_s} \ .$$

Data for the white dwarf star is

$$B = 1.33\times10^{22} \text{ Pa} \ ,$$

and

$$\rho = 1.00\times10^{10} \text{ kg m}^{-3} \ ,$$

the speed of sound will be

$$v_s = \sqrt{\frac{1.33\times10^{22}}{1.0\times10^{10}}} \text{ m s}^{-1} = 1.153\times10^{6} \text{ m s}^{-1}.$$

It is given that the typical radius of a white dwarf star is 0. 009 solar radius. Solar radius is 6.96×10^{8} m. So, we estimate its period of pulsation to be

$$T = \frac{4\times6.96\times10^{8}\times9\times10^{-3}}{1.15\times10^{6}} \text{ s} = 21.7 \text{ s}.$$