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Problem 20.9 (RHK)

A stone is dropped into a well. The sound of the splash is heard 3.00 s later. We have to find the depth of the well.

Solution:

Let the depth of the well be h m. As the stone is in free-fall, its acceleration will be g. The time t_1 taken by the stone to reach the bottom of the well is

$$\frac{gt_1^2}{2} = h,$$

or

$$t_1 = \sqrt{\frac{2h}{g}}.$$

The time t_2 taken by the sound of splash in travelling the length of the well is

$$t_2 = \frac{h}{v_s},$$

where v_s is the speed of sound in air and is taken to be 343 m s^{-1}.

The total time $t = t_1 + t_2$ is 3 s. We thus have the equation

$$\frac{h}{v_s} + \sqrt{\frac{2}{g}} \times \sqrt{h} = 3,$$

or

$$h + v_s \sqrt{\frac{2}{g}} \times \sqrt{h} - 3v_s = 0.$$

Roots of this quadratic equation are

$$\sqrt{h} = \frac{-v_s \sqrt{2/g} \pm \sqrt{\left(v_s \sqrt{2/g}\right)^2 + 12v_s}}{2}.$$

Substituting the values, we get

$$\sqrt{h} = \frac{-154.9 \pm \sqrt{24010 + 4116}}{2} = \frac{-154.9 \pm 167.7}{2}.$$

The physical solution is

$\sqrt{h} = 6.40$

and

$h = 40.9$ m.