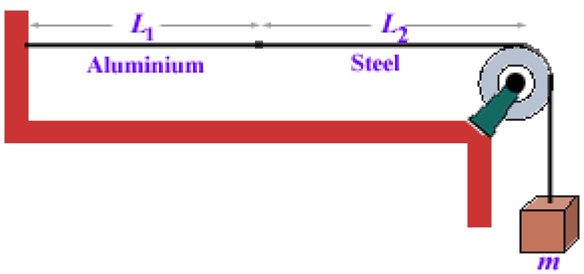
141.

<u>Problem 19.54 (RHK)</u>

An aluminium wire of length $L_1 = 60.0\ \text{cm}$ *and of cross-sectional area* $1.00\times10^{-2}\ \text{cm}^2$ *is connected to a steel wire of the same cross-sectional area. The compound wire, loaded with a block m of mass* 10.0 kg, *is arranged as shown in the figure so that the distance from the joint to the supporting pulley is* 86.6 cm. *Transverse waves are set up in the wire by using an external source of variable frequency.* (a) *We have to find the lowest frequency of excitation for which standing waves are served such that the joint in the wire is a node.* (b) *We have to find the total number of nodes observed at this frequency, excluding the two at the ends of the wire. The density of aluminium is* $2.60\ \text{g cm}^{-3}$ *and that of steel is* $7.80\ \text{g cm}^{-3}$.



Solution:

From the data of the problem we will first calculate the speeds of wave motion in the aluminium and steel wires. As the speed depends on the tension and the mass per unit length we will first compute these quantities. The tension in the wires that have the same cross-sectional area, πr^2, is equal to the weight of the block that is attached to one end of the wire through a pulley. It is

$$F = mg = 10 \times 9.8 \text{ N} = 98 \text{ N}.$$

Cross-sectional area of the two wires has been given to be

$$a = 1.00 \times 10^{-6} \text{ m}^2.$$

Density of aluminium, $\rho_{al} = 2.6 \times 10^3$ kg m^{-3}. Therefore, the mass per unit length of the aluminium wire μ_{al} is

$$\mu_{al} = \rho_{al} a = 2.6 \times 10^3 \times 10^{-6} \text{ kg m}^{-1} = 2.6 \times 10^{-3} \text{ kg m}^{-1}.$$

Density of steel, $\rho_{st} = 7.8 \times 10^3$ kg m^{-3}. Therefore, the mass per unit length of the steel wire μ_{st} is

$$\mu_{st} = \rho_{st} a = 7.8 \times 10^3 \times 10^{-6} \text{ kg m}^{-1} = 7.8 \times 10^{-3} \text{ kg m}^{-1}.$$

Velocity of wave motion in aluminium wire

$$v_{al} = \sqrt{\frac{F}{\mu_{al}}} = \sqrt{\frac{98}{2.6 \times 10^{-3}}} \text{ m s}^{-1} = 194.1 \text{ m s}^{-1}.$$

Velocity of wave motion in steel wire

$$v_{st} = \sqrt{\frac{F}{\mu_{st}}} = \sqrt{\frac{98}{7.8\times10^{-3}}} \text{ m s}^{-1} = 112.1 \text{ m s}^{-1}.$$

Length of the aluminium wire, $L_1 = 0.6$ m.

Length of the steel wire from the joint to the pulley, $L_2 = 0.866$ m.

Let the number of loops of the standing wave in the aluminium wire when its joint with the steel wire is a node be n_1. Then

$$n_1\left(\lambda_{al}/2\right) = 0.6 \text{ m},$$

and

$$\lambda_{al} = \frac{1.2}{n_1} \text{ m}.$$

Resonant frequency

$$f = \frac{v_{al}}{\lambda_{al}} = \frac{194.1\times n_1}{1.2} \text{ Hz} = 161.6 n_1 \text{ Hz}.$$

Let the number of loops of the standing wave in the steel wire from the joint with the aluminium wire and the pulley be n_2. Then

$$n_2\left(\lambda_{st}/2\right) = 0.866 \text{ m},$$

and

$$\lambda_{st} = \frac{1.732}{n_2} \text{ m}.$$

(a)

The resonant frequency for standing wave calculated from the data for the steel wire is

$$f = \frac{v_{st}}{\lambda_{st}} = \frac{112.1\times n_2}{1.732}\ \text{Hz} = 64.7\,n_2\ \text{Hz}.$$

We have to find minimum integers n_1 and n_2 which satisy the condition

$$161.6\,n_1 = 64.7\,n_2.$$

We note that this condition is satisfied for $n_1 = 2$ and $n_2 = 5$. The lowest frequency of excitation for which standing waves will be observed with the joint of the two wires being a node is

$$f = 161.6\times 2\ \text{Hz} \approx 64.7\times 5\ \text{Hz} = 323\ \text{Hz}.$$

(b)

The total number of nodes observed at this frequency, excluding the two at the ends of the wire, will be 6.