140.

Problem 19.41 (RHK)

The equation of a transverse wave travelling in a string is given by

$$y = 0.15\sin\left(0.79x - 13t\right),$$

where x and *y are expressed in meters and t is in seconds.*

We have to find (a) *the displacement at x* = 2.3 m, t = 0.16 s.

(b) *We have to write down the equation of a wave that, when added to the given one, would produce standing waves on the string.*

(c) *We have to find the displacement of the resulting standing wave at x* = 2.3 m, *t* = 0.16 s.

Solution:

We have been given the equation of a transverse travelling wave in a string. It is

$$y = 0.15\sin\left(0.79x - 13t\right),$$

where *x* and *y* are expressed in meters and *t* in seconds.

(a)

The displacement at $x = 2.3\ \text{m}$ and at $t = 0.16\ \text{s}$ will be

$$y = 0.15\sin\left(0.79\times 2.3 - 13\times 0.16\right)\text{m},$$
$$= 0.15\sin\left(1.817 - 2.08\right)\text{m} = -0.15\sin\left(0.263\right)\text{m},$$
$$= -0.15\times 0.259\ \text{m} = -0.0389\ \text{m} = -3.9\ \text{cm}.$$

(b)

Equation of the wave that when added to the given wave would produce standing waves in the string is given by an identical wave travelling in the opposite direction. It is

$$y' = 0.15\sin\left(0.79x + 13t\right).$$

Superposition of y and y',

$$y'' = y + y'$$

will produce a standing wave given by the expression

$$y''\left(x,t\right) = 2\times 0.15\sin\left(0.79x\right)\cos\left(13t\right).$$

(c)

The displacement of the resultant standing wave at $x = 2.3\ \text{m}$ and $t = 0.16\ \text{s}$ can be calculated from the equation of the standing wave. It is

$$y''\left(2.3, 0.16\right) = 0.30\sin\left(0.79\times 2.3\right)\cos\left(13\times 0.16\right)\ \text{m},$$
$$= 0.30\times\left(0.969\right)\times\left(-0.487\right)\ \text{m} = -0.14\ \text{m}.$$