

138.

Problem 19.55 (RHK)

A piano wire 1.4 m long is made of steel with density 7.8 g cm^{-3} and Young's modulus 220 MPa. The tension in the wire produces a strain of 1.0%. We have to calculate the lowest resonant frequency of the wire.

Solution:

Length of the steel wire = 1.4 m.

Density of steel, $\rho = 7.8 \text{ g cm}^{-3} = 7.8 \times 10^3 \text{ kg m}^{-3}$.

Mass per unit length of the wire, $\mu = \pi r^2 \rho$, where r is the radius of the wire.

Young's modulus of steel, $E = 220 \times 10^9 \text{ N m}^{-2}$.

Strain in the wire, $\Delta l/l = 0.01$.

Let the tension in the wire be F . Stress and strain are related by the modulus of elasticity as

$$E = \frac{F/\pi r^2}{\Delta l/l}.$$

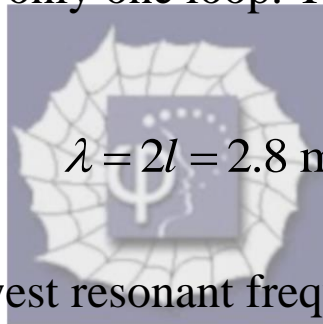
So we have

$$\frac{F}{\pi r^2} = 220 \times 10^9 \times \frac{1}{100} \text{ N m}^{-2} = 22 \times 10^8 \text{ N m}^{-2}.$$

Speed of wave propagation in a stressed wire is given by

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{\pi r^2 \rho}} = \sqrt{\frac{22 \times 10^8}{7.8 \times 10^3}} \text{ m s}^{-1} = 531 \text{ m s}^{-1}.$$

Wavelength of the longest standing wave corresponds to the configuration when the two ends of the wire are nodes and there is only one loop. That is



Therefore, the lowest resonant frequency of the wire

$$f = \frac{v}{\lambda} = \frac{531}{2.8} \text{ Hz} = 190 \text{ Hz}.$$