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<u>Problem 19.55 (RHK)</u>

A piano wire 1.4 m *long is made of steel with density* 7.8 g cm^{-3} *and Young's modulus* 220 MPa. *The tension in the wire produces a strain of* 1.0%. *We have to calculate the lowest resonant frequency of the wire.*

<u>Solution:</u>

Length of the steel wire = 1.4 m.

Density of steel, $\rho = 7.8 \text{ g cm}^{-3} = 7.8 \times 10^3 \text{ kg m}^{-3}$.

Mass per unit length of the wire, $\mu = \pi r^2 \rho$, where r is the radius of the wire.

Young's modulus of steel, $E = 220 \times 10^9 \text{ N m}^{-2}$.

Strain in the wire, $\Delta l / l = 0.01$.

Let the tension in the wire be F. Stress and strain are related by the modulus of elasticity as

$$E = \frac{F / \pi r^2}{\Delta l / l}.$$

So we have

$$\frac{F}{\pi r^2} = 220\times10^9 \times \frac{1}{100} \text{ N m}^{-2} = 22\times10^8 \text{ N m}^{-2}.$$

Speed of wave propagation in a stressed wire is given by

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{\pi r^2 \rho}} = \sqrt{\frac{22\times10^8}{7.8\times10^3}} \text{ m s}^{-1} = 531 \text{ m s}^{-1}.$$

Wavelength of the longest standing wave corresponds to the configuration when the two ends of the wire are nodes and there is only one loop. That is

$$\lambda = 2l = 2.8 \text{ m}.$$

Therefore, the lowest resonant frequency of the wire

$$f = \frac{v}{\lambda} = \frac{531}{2.8} \text{ Hz} = 190 \text{ Hz}.$$