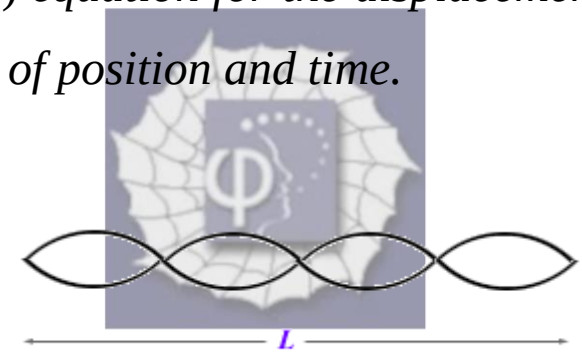
135.

Problem 19.43 (RHK)

Vibrations from a 622-Hz *tuning fork set up standing waves in a string clamped at both ends. The wave speed for the string is* 388 m s^{-1}. *The standing wave has four loops and is formed by superposition of waves of amplitude* 1.90 mm. *We have to find* (a) *the length of the string;* (b) *equation for the displacement of the string as a function of position and time.*



Solution:

Frequency of the tuning fork, $f = 622$ Hz.

Wave speed for the string, $v = 388$ m s^{-1}.

Travelling waves generated in the string by the vibrations of the tuning fork superpose to produce standing waves. Wavelength λ and frequency *f* determine the speed *v*,

$$\lambda = \frac{v}{f} = \frac{388}{622}\ \text{m} = 0.624\ \text{m}\ .$$

As the standing wave in the string has four loops, the length of the string $L = 2\lambda$.

Therefore, L = 1.248 m.

Equation of standing waves formed out of travelling wave given by the functions

$$y_1 = y_m \sin k\left(x - vt\right)$$

and

$$y_2 = y_m \sin k\left(x + vt\right)$$

is given by

$$y = y_1 + y_2.$$

It is

$$y = \left(2y_m \sin kx\right)\cos 2\pi ft \ .$$

We now work out the numerical values of variables in the above function.

$$2y_m = 3.80 \times 10^{-3} \text{ m};$$

and

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.624} \text{ m}^{-1};$$

and

$$2\pi f = 2\pi \times 622 = 3908.$$

The equation of the standing wave is

$$y = 3.80 \times 10^{-3} \sin\left(10.1x\right)\cos\left(3908t\right) \text{ m},$$

where x and y are in meter and t is in second.