132.

Problem 17.43P (HRW)

Three sinusoidal waves of the same frequency travel along a string of in the positive direction of an x axis. Their amplitudes are $y_1, y_1/2$, *and* $y_1/3$, *and their phase constants are* 0, $\pi/2$ *and* π, *respectively. We have to find the amplitude and the phase constant of the resultant wave.*

Solution:

From the data of the problem we can write the functions for the three waves as

$$\psi_1 = y_1 \sin(kx - \omega t),$$

$$\psi_2 = y_1/2 \sin(kx - \omega t + \pi/2) = y_1/2 \cos(kx - \omega t),$$

and

$$\psi_3 = y_1/3 \sin(kx - \omega t + \pi) = -y_1/3 \sin(kx - \omega t).$$

By the principle of superposition the resultant wave will be described by the function

$$\psi = \psi_1 + \psi_2 + \psi_3$$

$$= y_1 \sin(kx - \omega t) + \frac{y_1}{2}\cos(kx - \omega t) - \frac{y_1}{3}\sin(kx - \omega t)$$

$$= \frac{2y_1}{3}\sin(kx - \omega t) + \frac{y_1}{2}\cos(kx - \omega t).$$

We define

$$\sin\theta = \frac{1/2}{\left((1/2)^2 + (2/3)^2\right)^{1/2}} = \frac{3}{5},$$

and

$$\cos\theta = \frac{2/3}{\left((1/2)^2 + (2/3)^2\right)^{1/2}} = \frac{4}{5}.$$

The resultant wave can thus be expressed by the function

$$\psi = \frac{5}{6} y_1 \sin(kx - \omega t + \theta).$$

Therefore, the amplitude of the resultant wave is

$$a = \frac{5}{6} y_1 = 0.83 y_1,$$

and its phase constant is

$$\theta = \sin^{-1} 3/5 = 36.8^0.$$