130.

Problem 19.38 (RHK)

This problem uses the result of the problem 129 (19.38 RHK). *Let us suppose that d* = 230 km *and H* = 510 km. *The waves are* 13.0 MHz *radio waves* ($v = 3.00 \times 10^8$ m s^{-1}). *At the detector D the combined signal strength varies from a maximum to zero and back to a maximum again six times in* 1 min. *We have to find the speed with which the reflecting layer is moving. We can assume that the layer is moving slowly, so that the vertical distance moved in* 1 min *is small compared to H and d.*

Solution:

Data of the problem are

$d = 230$ km,

$H = 510$ km,

$f = 13.0 \times 10^6$ Hz,

and v the speed of the wave is $v = 3.00 \times 10^8$ m s^{-1}. Therefore, the wavelength λ of the wave is

$$\lambda = \frac{3.0\times10^{8}\ \text{m s}^{-1}}{13.0\times10^{6}\ \text{s}^{-1}} = 23\ \text{m.}$$

In the problem it is given that the reflecting layer is moving with speed such that the signal strength at the detector varies from maximum to zero and back to maximum again six times in a minute. Let *h* be the distance moved by the reflecting layer in 1 min. This implies that

$$6\lambda = 2\left(\sqrt{d^2+4\left(H+h\right)^2}-\sqrt{d^2+4H^2}\right).$$

We have been given that the vertical distance h is small compared to *H* and *d*. In this approximation we use the result that

$$\left(d^2+4\left(H+h\right)^2\right)^{1/2} \approx \left(d^2+4H^2\right)^{1/2} + \frac{4Hh}{\left(d^2+4H^2\right)^{1/2}}.$$

In this approximation, we have the result

$$\frac{4Hh}{\left(d^2+4H^2\right)^{1/2}} = 3\lambda.$$

Substituting the data of the problem, we find

$$\frac{4 \times 510h}{\left(230^2 + 4 \times 510^2\right)^{1/2}} = 3 \times 23 \text{ m},$$

or

$$h = 35.4 \text{ m}.$$

Thus we find that the reflecting layer is moving with a speed of 35.4 m min^{-1}.