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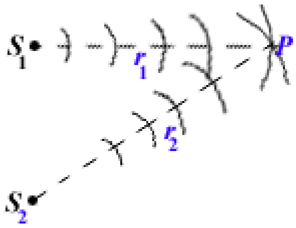
Problem 19.36 (RHK)

We consider two point sources S_1 *and* S_2, *which emit waves of the same frequency and amplitude. The waves start in the same phase, and this phase relation at the sources is maintained throughout time. We consider points P at which* r_1 *is nearly equal to* r_2. *We have to show* (a) *that the superposition of these two waves gives a wave whose magnitude* y_m *varies with the position P approximately according to*

$$y_m = \frac{2Y}{r}\cos\frac{k}{2}\left(r_1 - r_2\right),$$

in which $r = \left(r_1 + r_2\right)/2$. (b) *We then have to show that total cancellation occurs when* $r_1 - r_2 = \left(n + \frac{1}{2}\right)\lambda$, *n being any integer, and the total re-enforcement occurs when* $r_1 - r_2 = n\lambda$. *The locus of points whose difference in distance from two fixed points is a constant is a hyperbola, the fixed points being the foci. Hence each value of n gives a hyperbolic line of constructive interference and a hyperbolic line of destructive* *interference. At points at which r_1 and r_2 are not approximately equal (as near the sources), the amplitudes of the waves from S_1 and S_2* differ *and cancellations are only partial.*



Solution:

We will write functions representing spherical waves of the same frequency and amplitude emitted by sources S_1 and S_2. We assume that the waves from S_1 and S_2 start in the same phase and this relation is maintained throughout time. These functions are

$$y_1 = \frac{Y}{r_1}\sin k\left(r_1 - vt\right),$$
$$y_2 = \frac{Y}{r_2}\sin k\left(r_2 - vt\right),$$

where r_1 and r_2 are distances measured from S_1 and S_2, respectively. The resultant wave at any point in space will be given by the superposition of these two spherical waves. If at some point P the distances r_1 and r_2 are approximately equal, we may approximate Y/r_1 and Y/r_2 by $\dfrac{Y}{(r_1+r_2)/2}$.

In this approximation,

$$y = y_1 + y_2 = \frac{Y}{(r_1+r_2)/2}\left(\sin k\left(r_1 - vt\right) + \sin k\left(r_2 - vt\right)\right)$$

Or,

$$y = \frac{2Y}{r}\cos\frac{k}{2}\left(r_1 - r_2\right)\sin k\left(r - vt\right),$$

where

$$r = \frac{r_1 + r_2}{2}.$$

We rewrite the resultant wave function in the form

$$y = y_m \sin k\left(r - vt\right).$$

Amplitude y_m varies with position *P* approximately as

$$y_m = \frac{2Y}{r}\cos\frac{k}{2}\left(r_1 - r_2\right).$$

From this expression we note that $y_m = 0$, for

$$\left\{\begin{array}{l} \dfrac{k}{2}\left(r_1 - r_2\right) = \left(n + \tfrac{1}{2}\right)\pi, \text{ or} \\ \left(r_1 - r_2\right) = \left(n + \tfrac{1}{2}\right)\lambda \end{array}\right\}, \; n \text{ being any integer.}$$

In this situation there is total cancellation of the waves reaching at P from sources S_1 and S_2.

And, at points where

$$\frac{k}{2}\left(r_1 - r_2\right) = n\pi,\ \left(k = 2\pi/\lambda\right),$$

or where

$$\left(r_1 - r_2\right) = n\lambda,$$

the two waves reinforce each other and there is constructive interference.

The locus of points whose difference in distance from two fixed points is a constant describes a hyperbola, the fixed points being foci. Hence each value of *n* gives a hyperbolic line of constructive interference and a hyperbolic line of destructive interference.