127.

Problem 19.27 (RHK)

A wave travels out uniformly in all directions from a point source. (a) *We have to justify that the expression for the displacement y of the medium at any distance r from the from the source is*

$$y = \frac{Y}{r} \sin k\left(r - vt\right).$$

We have to consider the speed, direction of propagation, periodicity, and intensity of the wave.

(b) *We have to find the dimension of the constant Y.*

Solution:

(a)

The intensity, *I*, of a spherical wave at a distance r from its source has to vary as $1/r^2$, because $I \times \left(4\pi r^2\right)$, which is the total amount of energy flowing per second through a spherical surface of radius *r* centred at the source, is equal to the power of the source and is therefore a number independent of the variable r. We know that the intensity of a wave is proportional to the modulus square of its amplitude.

We will justify that the function

$$y = \frac{Y}{r}\sin k\left(r - vt\right).$$

describes spherical waves. As the function $\sin k\left(r - vt\right)$ is of the generic form $f\left(r - vt\right)$ it represents wave motion along the direction of increasing *r*. From this function we note that the speed of the wave is *v* and the frequency of the wave $\omega = vk$, where *k* the wave number of the wave determines the wavelength $\lambda = 2\pi/k$. As the intensity of wave motion is proportional to the square of the amplitude, which for the function is Y^2/r^2, where *Y* is a constant, it describes spherical wave propagation.

(b)

As *y* represents the displacement of the medium, its dimension has to be that of length $\left[\mathrm{L}\right]$. Therefore, the dimension of the constant *Y* is $\mathrm{length}^2 \left[\mathrm{L}\right]^2$.