125.

Problem 17.35P (HRW)

A transverse sinusoidal wave is generated at one end of a long, horizontal string by a bar that moves up and down through a distance of 1.00 cm. *The motion is continuous and is repeated regularly* 120 *times per second. The string has linear density* 120 g/m *and is kept under a tension of* 90.0 N.

We have to find (a) *the maximum value of the transverse speed u and* (b) *the maximum value of the transverse component of the tension.* (c) *We have to show that the two maximum values calculated above occur at the same phase values for the wave. We have to find transverse displacement y of the string at these phases.* (d) *We have to find the maximum rate of energy transfer along the string.* (e) *We have to find the transverse displacement y when the maximum transfer occurs.* (f) *We have to find the minimum rate of energy transfer along the string.* (g) *We have to find the transverse displacement y when this minimum transfer occurs.*

Solution:

As the transverse sinusoidal wave is being generated by moving one end of the sting up and down 120 times per second, the frequency, *f*, of the wave is

$$f = 120 Hz \ ,$$

and the angular frequency ω is

$$\omega = 2\pi f = 754 \text{ rad s}^{-1} \ .$$

The linear mass density, μ, of the string is

$$\mu = 120 \times 10^{-3} \text{ kg m}^{-1} \ .$$

As the string is kept under a tension F = 90.0 N, the speed of transverse wave motion is

$$v = \sqrt{\frac{90.0}{120 \times 10^{-3}}} \text{ m s}^{-1} = 27.4 \text{ m s}^{-1}.$$

Wave number k of the sinusoidal wave is

$$k = \frac{\omega}{v} = \frac{754}{27.4} \text{ m}^{-1} = 27.5 \text{ m}^{-1} \ .$$

As the wave is generated by moving one end of the string up and down through a distance of 1.00 cm, the amplitude of transverse oscillation of the string, *a*, is

$$a = 0.5 \times 10^{-2} \text{ m} .$$

We now have all the parameters that determine the equation for the sinusoidal wave motion

$$y = a \sin(\omega t - kx) .$$

Instantaneous transverse speed of oscillation of the string at position x is given by

$$u = \frac{dy}{dt} = a\omega \cos(\omega t - kx) .$$

(a)

We note from the function giving speed of transverse motion of the string that its maximum value is

$$u_{\max} = aw = 0.5 \times 10^{-2} \times 754 \text{ m s}^{-1} = 3.77 \text{ m s}^{-1} .$$

(b)

The transverse component of the tension in the string at position x and at time t is given by $-F \sin\theta$, which for small displacements can be approximated by $-F \tan\theta$. We thus have

$$F_y = -F \frac{dy}{dx} = +Fak \cos(\omega t - kx) .$$

We note that the maximum value of the transverse component of the tension in the string is

$$\left(F_y\right)_{\max} = Fak = 90\times 0.5\times 10^{-2}\times 27.5\ \text{N} = 12.4\ \text{N}.$$

(c)

We note that these two maximum values occur for the same phase value, $\omega t - kx = 0$.

The transverse displacement of the string at this phase is zero.

(d)

The power transferred along the string is given by

$$P\left(x,t\right) = uF_y = a^2\omega kF\cos^2\left(\omega t - kx\right)\ .$$

Therefore, the maximum power transferred along the string is

$$P_{\max} = a^2\omega kF = \left(0.5\times 10^{-2}\right)^2\times 754\times 27.5\times 90\ \text{W} = 46.6\ \text{W}.$$

(e)

At this phase the transverse displacement of the string is zero.

(f)

Minimum power transferred is zero. It occurs when phase $\omega t - kx = \pi/2, 3\pi/2$.

(g)

The transverse displacement of the string at these phase is maximum and is $\pm a = \pm 0.5 \times 10^{-2}$ m.