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Problem 19.23 (RHK)

A non-uniform wire of length L and mass M has a variable linear mass density given by $\mu = kx$ *, where x is the distance from one end of the wire and k is a constant. We have to show* (a) *that* $M = kL^2/2$. (b) *We have to show that the time t required for a pulse generated at one end of the wire to travel to the other end is given by* $t = \sqrt{8ML/9F}$ *, where F is the tension in the wire.*

Solution:

(a)

Mass density of the wire is $\mu = kx$. Therefore, mass of the wire is

$$M = \int_0^L kx dx = \frac{1}{2} kL^2.$$

(b)

Speed of wave motion in the wire will be a function of position *x* as mass density varies with *x*. As the wire is under uniform tension *F*, we have

$$v(x) = \sqrt{\frac{F}{kx}} \ .$$

Therefore, the time t required by a wave pulse generated at one end to travel to the other end of the wire will be

$$t = \int_0^L \frac{dx}{v(x)} = \int_0^L \frac{dx\sqrt{kx}}{\sqrt{F}} = \frac{2}{3} L^{3/2} \sqrt{\frac{k}{F}} = \sqrt{\frac{4}{9} L^3 \times \frac{2M}{L^2 F}} = \sqrt{\frac{8ML}{9F}} \ .$$