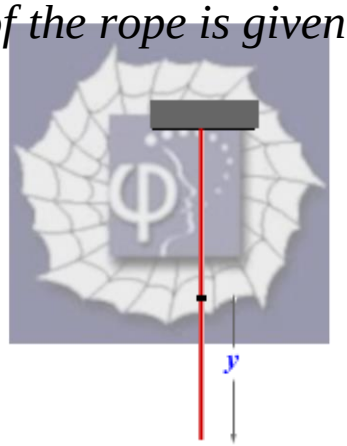
122.

Problem 17.32P (HRW)

A uniform rope of mass m and length l hangs from a ceiling. We have to show (a) *that the speed of a transverse wave on the rope is a function of y, the distance from the lower end, and is given by* $v = \sqrt{gy}$. (b) *We have to show that the time a transverse wave takes to travel the length of the rope is given by* $t = 2\sqrt{l/g}$.



Solution:

(a)

Speed of transverse waves on the sting will be a function of *y*, the distance on the string measured from its bottom end, because the tension in the string is determined by the weight of the string hanging below the level *y*.

Therefore,

$$T(y) = \frac{m}{l} \times yg,$$

where *m* is the mass and *l* is the length of the string. As speed of a transverse wave in a string is given by the relation

$$v = \sqrt{\frac{T}{\mu}},$$

where T is the tension and μ is the mass per unit length of the string. We thus have the result

$$v(y) = \sqrt{\frac{T(y)}{m/l}} = \sqrt{\frac{gy\,m/l}{m/l}} = \sqrt{gy}\ .$$

(b)

We will next calculate the time taken be a transverse wave for travelling the length of the string

$$dt = \frac{dy}{v(y)} = \frac{dy}{\sqrt{gy}}\ .$$

By integrating this equation, we find

$$t = \int_0^l \frac{dy}{\sqrt{gy}} = \frac{1}{\sqrt{g}} \times \left[\frac{y^{1/2}}{1/2} \right]_0^l = 2\sqrt{\frac{l}{g}}\ .$$