121.

Problem 17.29P (HRW)

We consider the arrangement in which two strings of different linear mass density hold a mass through pulleys as shown in the figure 1. *String* 1 *has linear density of* 3.00 g/m *and string* 2 *has a linear density of* 5.00 g/m. *They are under tension owing to the hanging block of mass* M = 500 g. *We have to calculate* (a) *the wave speed in each string.* (b) *In the situation when the block is divided into two blocks* (*with* $M = M_1 + M_2$) *and the apparatus is rearranged as shown in the figure* 2. *We have to find* M_1 *and* M_2 *such that the wave speeds in the two strings are equal.*

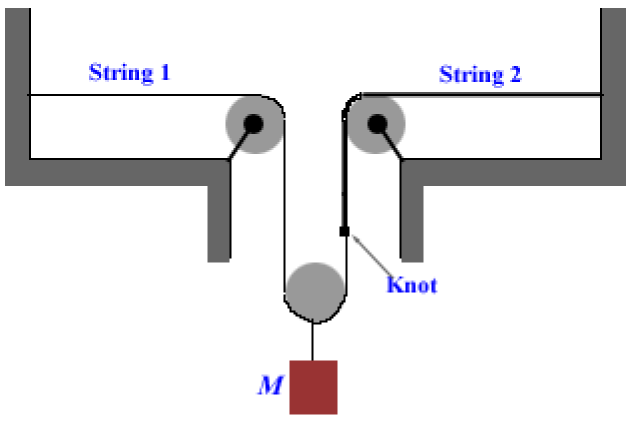

Figure 1

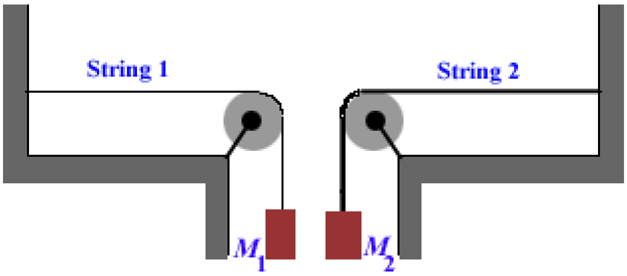

Figure 2

Solution:

(a)

Speed of wave motion in a taut string is given be the relation

$$v = \sqrt{\frac{T}{\mu}},$$

where T is the tension in the string and μ is its mass per unit length.

In solving the first part of the problem we will use the following data

Mass per unit length of string 1, $\mu_1 = 3.0 \times 10^{-3}$ kg m^{-1},

and

Mass per unit length of string 2, $\mu_2 = 5.0 \times 10^{-3}$ kg m^{-1}.

Tension in each of the strings

$T = \frac{1}{2} \times 0.5 \times 9.8 \text{ N} = 2.45 \text{ N}.$

Using the result that wave speed in a string,

$$v = \sqrt{\frac{T}{\mu}} \ ,$$

we find that the wave speed in string 1

$$v_1 = \sqrt{\frac{2.45}{3 \times 10^{-3}}} \text{ m s}^{-1} = 28.6 \text{ m s}^{-1}.$$

And , wave speed in string 2

$$v_2 = \sqrt{\frac{2.45}{5.0 \times 10^{-3}}} \text{ m s}^{-1} = 22.1 \text{ m s}^{-1}.$$

(b)

We next consider the situation when the block of mass 500 g is to be so divided into two pieces, which when hanged with strings 1 and 2 independently will result in equal wave speeds in both the strings. This is equivalent to the condition

$$\sqrt{\frac{M_1 g}{\mu_1}} = \sqrt{\frac{M_2 g}{\mu_2}} \ ,$$

or

$$\frac{M_1}{\mu_1} = \frac{M - M_1}{\mu_2} \ .$$

We find,

$$M_1 = \frac{M\mu_1}{\mu_1 + \mu_2} = \frac{500 \times 3}{(3+5)} \text{ g} = 187.5 \text{ g}$$

and

$$M_2 = 312.5 \text{ g.}$$