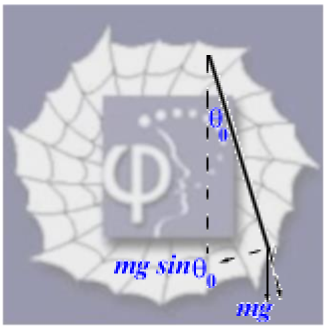
120.

<u>Problem 15.55 (RHK)</u>

A simple pendulum of length L and mass m is suspended in a car that is travelling with a constant speed v around a circle of radius R. Pendulum undergoes small oscillations in a radial direction about its equilibrium position. We have to find the frequency of oscillation.



<u>Solution:</u>

It is given that a bob with a string is hanging in a car that is travelling with a constant speed *v* around a circle of radius *R*. The required centripetal force is obtained by the incline of the string attached to the bob by angle θ_0 from the vertical. The component of the weight of the bob perpendicular to the string $mg\sin\theta_0$ provides the centripetal force, that is

$$mg\sin\theta_0 = \frac{mv^2}{R} \ ,$$

or

$$\sin\theta_0 = \frac{v^2}{Rg} \ .$$

Let the bob be displaced by a small angle θ. We will show that for small oscillation the bob will execute simple harmonic motion as a simple pendulum.

When the pendulum is displaced by an additional angle θ restoring force on the bob will be

$$mg\sin(\theta_0 + \theta).$$

Let us consider motion in a uniformly rotating frame with angular speed $\omega = v/R$. Let the length of the string with which the bob is hanging be L. Equation of motion of the bob in this frame of reference will be

$$mL\frac{d^2\theta}{dt^2} - \frac{mv^2}{R} + mg\sin(\theta_0 + \theta) = 0.$$

Under the approximation

$\sin\theta \approx \theta$, and $\cos\theta \approx 1$,

equation of motion reduces to the form

$$L\frac{d^2\theta}{dt^2} - \frac{v^2}{R} + g(\sin\theta_0 + \theta\cos\theta_0) = 0 \ .$$

Using the result

$$g\sin\theta_0 = \frac{v^2}{R} \ ,$$

equation of motion simplifies to the form

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\left(1-\sin^2\theta_0\right)^{1/2}\theta = 0.$$

It is an equation of SHM. Frequency of oscillation is

$$\frac{2\pi}{T} = \sqrt{\frac{g}{L}\left(1-\left(\frac{v^2}{Rg}\right)^{1/2}\right)} \ ,$$

where T is the period of oscillation. Frequency ν of the pendulum is

$$\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{L}\left(1-\left(\frac{v^2}{Rg}\right)^{1/2}\right)} \ .$$