119.

Problem 15.47 (RHK)

A physical pendulum consists of a meter stick that is pivoted at a small hole drilled through the stick a distance x from the 50.0-cm *mark. The period of oscillation is observed to be* 2.50 s. *We have to find the distance x.*

Solution:

Let the mass of the meter stick be *m*. We denote its length, which is 1.0 m, by *L*. The rotational inertia of the stick about the axis passing through its centre of mass is $mL^2/12$. Therefore, its rotational inertia about the axis passing through the pivot, which is at a distance x from the centre of the meter stick, will be

$$I = \frac{mL^2}{12} + mx^2 \ .$$

Period of the physical pendulum is given by the expression

$$T = 2\pi\sqrt{\frac{I}{mgx}}\ ,$$

or

$$T^2 mgx = 4\pi^2\left(\frac{mL^2}{12} + mx^2\right),$$

or

$$x^2 - \frac{T^2 g}{4\pi^2}x + \frac{L^2}{12} = 0\ .$$

Substituting L = 1.0 m and T= 2.50 s, we have the quadratic equation

$$x^2 - 1.55x + \tfrac{1}{12} = 0\ .$$

Roots of this quadratic equation are

$$x = 5.5 \text{ cm}, 1.5 \text{ m}.$$

As we are considering oscillations of a meter rod, the physical solution is $x = 5.5$ cm.