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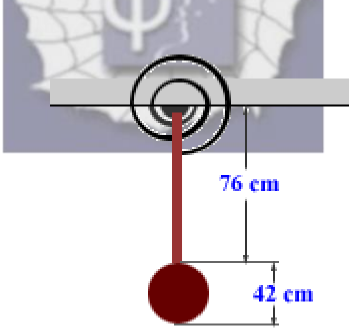
<u>**Problem 16.81P (HRW)**</u>

A 2.5 kg *disk,* 42 cm *in diameter, is supported by a massless rod,* 76 cm *long, which is pivoted at its end. We have to find* (a) *the period of oscillation when the massless torsion string is not connected.* (b) *When the torsion spring is connected so that, in equilibrium, the rod hangs vertically, we have to find the torsional constant of the spring so that the new period of oscillation is* 0.50 s *shorter than before.*



<u>**Solution:**</u>

(a)

In the first part of the problem we will consider oscillations of the pendulum when the torsion spring does not touch the rod connecting the disk with the pivot.

In the approximation of negligible mass of the rod compared to the mass of the disk, the rotational inertia of the physical pendulum about the axis of oscillation passing through the pivot will be

$$I = \tfrac{1}{2} MR^2 + Md^2 \ ,$$

where M is the mass and R is the radius of the disk, and d is the distance of the centre of the disk from the pivot. Data of the problem are

$$\begin{aligned} d &= (76 + 21) \text{ cm} = 0.97 \text{ m}, \\ R &= 0.21 \text{ m}, \\ M &= 2.5 \text{ kg}. \end{aligned}$$

Period of oscillation of a physical pendulum is given by the mathematical expression

$$T = 2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{\left(d^2 + \tfrac{1}{2} R^2\right)}{gd}} \ ,$$

$$= 2\pi\sqrt{\frac{\left(0.97^2 + \tfrac{1}{2}\times 0.21^2\right)}{9.8\times 0.97}} = 2.0 \text{ s.}$$

(b)

We will next consider the physical pendulum when torsion spring is in contact with the rod. Let the torsional constant of the spring be θ. Equation of motion of the physical pendulum in this situation will be

$$I\frac{d^2\theta}{dt^2}+Mgd\theta+\tau\theta=0 \ .$$

It is an equation of SHM. Period T' of oscillations of the physical pendulum attached to a torsional spring will therefore be

$$\frac{2\pi}{T'}=\sqrt{\frac{Mgd+\tau}{I}} \ ,$$

or

$$T'=2\pi\sqrt{\frac{I}{Mgd+\tau}} \ .$$

We thus have the relation

$$Mgd+\tau=\frac{4\pi^2 I}{T'^2} \ ,$$

or

$$\tau=M\left(\frac{4\pi^2\left(d^2+\frac{1}{2}R^2\right)}{T'^2}-gd\right) \ .$$

From this result we can compute the value of τ when $T'=1.5$ s.

We find

$$\tau = 2.5\left(\frac{4\pi^2\left(0.97^2 + \tfrac{1}{2}\times 0.21^2\right)}{1.50^2} - 9.8\times 0.97 \right) \text{ N m rad}^{-1},$$

$$= 18.5 \text{ N m rad}^{-1} \, .$$