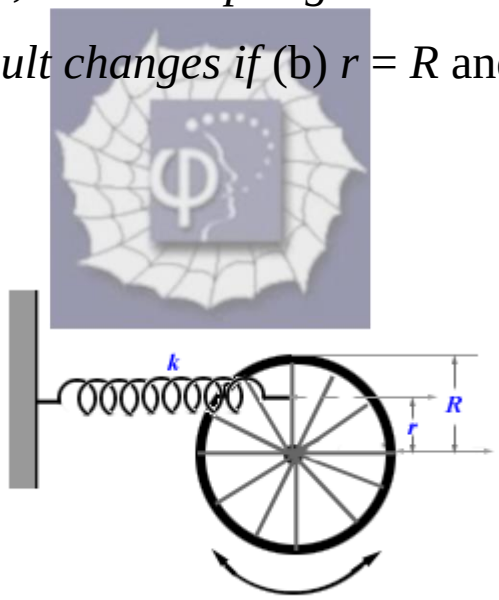
117.

Problem 16.80P (HRW)

A wheel is free to rotate about its fixed axle. A spring is attached to one of its spokes a distance r from the axle (*see the diagram*). *Assuming that the wheel is a hoop of mass m and radius R, we have to obtain* (a) *the angular frequency of small oscillations of this system in terms of m, R, r, and the spring constant k. We have to find how the result changes if* (b) $r = R$ and (c) $r = 0$.



Solution:

(a)

Rotational inertia of a hoop of mass *m* and radius *R* is $I = mR^2$. Let us assume that because of the spring attached to a spoke at distance *r* from the axle hoop

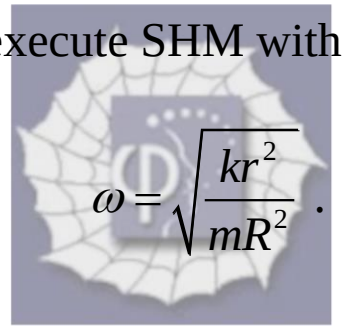
undergoes small oscillations. As the force constant of the spring is *k*, when the hoop has rotated clockwise by angle θ it will be pulled by the spring with force $kr\theta$. Torque on the hoop due to this force will be

$$\tau = -kr^2\theta \ .$$

So the equation of motion of the hoop will be

$$mR^2 \frac{d^2\theta}{dt^2} + kr^2\theta = 0 \ .$$

So the hoop will execute SHM with angular frequency



$$\omega = \sqrt{\frac{kr^2}{mR^2}} \ .$$

(b)

When *r* = *R*, angular frequency of SHM will be

$$\omega_0 = \sqrt{\frac{k}{m}} \ .$$

(c)

when r = 0, hoop will rotate freely without any acceleration.