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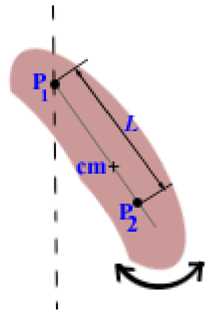
<u>Problem 15.53 (RHK)</u>

A physical pendulum has two possible pivot points; one has a fixed position and the other is adjustable along the length of the pendulum. The period of the pendulum when suspended from the fixed pivot is T. The pendulum is then reversed and suspended from the adjustable pivot. The position of this pivot is moved until, by trial and error, the pendulum has the same period as before, namely T. We have to show that the free-fall acceleration is given by

$$g = \frac{4\pi^2 L}{T} \ ,$$

in which L is the distance between the two pivot points. We note that g can be measured in this way without needing to know the rotational inertia of the pendulum or any of its other dimensions except L.



Solution:

A physical pendulum has two possible pivot points P_1 and P_2 ; point P_1 has a fixed position and P_2 is adjustable along the length of the pendulum. Let the period of oscillation of the pendulum when suspended from P_1 be T. Let d be the distance between P_1 and the centre of mass (cm) of the pendulum. Let I_{cm} be the rotational inertia of the physical pendulum about the axis passing through the cm and perpendicular to the plane of oscillation. The rotational inertia of the pendulum about an axis perpendicular to the plane of oscillation and passing through the pivot P_1 will be

$$I_{P_1} = I_{cm} + Md^2 \ ,$$

and its period of oscillation will be given by the relation

$$T = 2\pi\sqrt{\frac{I_{P_1}}{Mgd}} \ .$$

The period of oscillation when the physical pendulum oscillates about the pivot P_2 will be

$$T' = 2\pi\sqrt{\frac{I_{P_2}}{Mg\left(L-d\right)}} \ ,$$

$$I_{P_2} = I_{cm} + M\left(L-d\right)^2 \ .$$

L is the distance between P_1 and P_2 . L is to be so adjusted such that

$$T = T' \ .$$

This condition gives the equation that fixes I_{cm} in terms of L, g and T. It is

$$2\pi\sqrt{\frac{\left(I_{cm} + Md^2\right)}{Mgd}} = 2\pi\sqrt{\frac{\left(I_{cm} + M\left(L-d\right)^2\right)}{Mg\left(L-d\right)}} \ ,$$

or

$$\frac{I_{cm} + Md^2}{d} = \frac{I_{cm} + M\left(L-d\right)^2}{\left(L-d\right)} \ .$$

Simplifying this algebraic equation, we find

$$I_{cm} = Md\left(L-d\right).$$

Substituting this expression for I_{cm} in the expression for the period T, we get the result

$$T = 2\pi\sqrt{\frac{MdL - Md^2 + Md^2}{Mgd}} = 2\pi\sqrt{\frac{L}{g}} \ ,$$

or

$$g = \frac{4\pi^2 L}{T^2}.$$