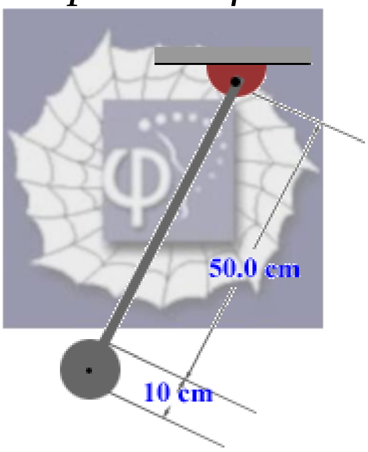
115.

Problem 16.67E (HRW)

A pendulum consists of a uniform disk with radius 10.0 cm *and mass* 500 g *attached to a uniform rod with length* 500 mm *and mass* 270 g. *We have to calculate* (a) *the rotational inertia of the pendulum about the pivot;* (b) *the distance between the pivot and the centre of mass of the pendulum;* (c) *the period of oscillation.*



Solution:

(a)

Distance of the centre of the disk from the pivot, $d = 60.0$ cm. Mass of the disk $M = 500$ g and its radius $R = 10.0$ cm. Therefore the rotational inertia of the disk about the pivot will be

$$I_{disk} = \frac{1}{2}MR^2 + Md^2 = 500\left(0.5\times100+3600\right) \text{ g cm}^2,$$

$$= 0.1825 \text{ kg m}^2 \ .$$

Rotational inertia of a rod of length $L = 50$ cm and mass $m = 270$ g about one of its ends is

$$I_{rod} = \frac{1}{3}mL^2 = \frac{1}{3}\times270\times50^2 \text{ g cm}^2 \ ,$$

$$= 0.0225 \text{ kg m}^2 \ .$$

Therefore, the rotational inertia of the physical pendulum about the pivot

$$I = I_{disk} + I_{rod} = \left(0.1825+0.0225\right) \text{ kg m}^2 = 0.205 \text{ kg m}^2.$$

(b)

Using the definition of the CM and measuring distances from the pivot, the distance of the CM of the pendulum from the pivot is

$$x_{cm} = \frac{500\times60+270\times25}{500+270} \text{ cm} = 47.7 \text{ cm}.$$

(c)

Period of oscillation of the pendulum

$$T = 2\pi\sqrt{\frac{I}{(M+m)gx_{cm}}} = 2\pi\sqrt{\frac{0.205}{0.77\times 9.8\times 0.477}} = 1.5 \text{ s.}$$