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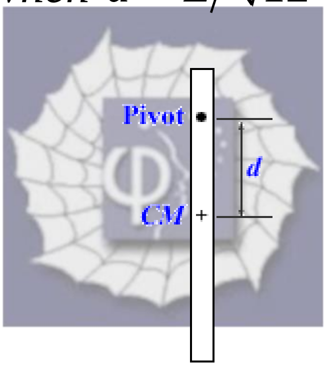
<u>Problem 15.49 (RHK)</u>

A pendulum is formed by pivoting a long thin rod of length L and mass m about a point on the rod which is a distance d above the centre of the rod. We have to find (a) *the small-amplitude period of this pendulum in terms of d, L, m, and g.* (b) *We have to show that the period has a minimum value when* $d = L/\sqrt{12} = 0.289L$.



<u>Solution:</u>

It is given that a pendulum has been formed by pivoting a long thin rod of length L and mass m about a point distance d from the centre of the rod. The rotational inertia of the rod about its centre is $mL^2/12$. Using the parallel axis theorem the rotational inertia of the rod about an axis passing through the pivot is found to be

$$I = \frac{1}{12} mL^2 + md^2 \ .$$

The small-amplitude period of SHM of this rod will be

$$T = 2\pi \sqrt{\frac{I}{mgd}} = 2\pi \sqrt{\frac{\left(\frac{1}{12} mL^2 + md^2\right)}{mgd}},$$

$$= 2\pi \sqrt{\frac{\left(\frac{1}{12} L^2 + d^2\right)}{gd}} \ .$$

Equivalently, we have

$$\frac{T^2}{4\pi^2} = \left(\frac{L^2}{12} + d^2\right) \times \frac{1}{gd} \ .$$

We will find the minimum value of *T*, by obtaining the equation of extremum

$$\frac{d\left(T^2 g / 4\pi\right)}{d'd'} = 0 \ ,$$

or

$$\left(\frac{L^2}{12} + d^2\right) \times \left(\frac{-1}{d^2}\right) + 2 = 0 \ .$$

Its solutions is

$$\frac{L^2}{12d^2} = 1 ,$$

or

$$d = \frac{L}{\sqrt{12}} = 0.289L .$$