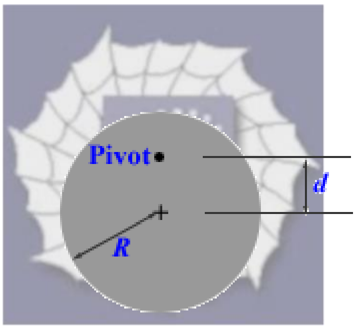
113.

Problem 15.45 (RHK)

A physical pendulum consists of a uniform solid disk of mass M = 563 g *and radius* R =14.4 cm *supported in a vertical plane by a pivot located a distance* d = 10.2 cm *from the centre of the disk. The disk is displaced by a small angle and released. We have to find the period of the resulting simple harmonic motion.*



Solution:

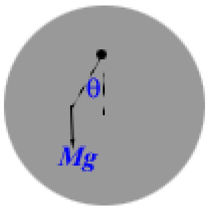
The centre of mass of the physical pendulum will be at the centre of the circular disk. Rotational inertia of a uniform disk of mass M and radius R about the axis passing through its CM and perpendicular to the plane of the disk is

$$I_{cm} = \frac{1}{2} MR^2 \ .$$

By the parallel axis theorem the rotational inertia of the disk about the axis passing through the pivot will be

$$I_p = \frac{1}{2}MR^2 + Md^2 \ .$$

Let us consider small oscillations of the physical pendulum about its pivot.



Oscillating physical pendulum

Let at some instant the line joining the pivot to the CM be at angle θ from the vertical. Torque exerted by the weight of the pendulum about the axis passing through the CM will be

$$\tau = -Mgd\sin\theta.$$

As we are considering small oscillations, for small angles we can approximate

$$\sin\theta \approx \theta \ .$$

Equation of motion of the physical pendulum in this approximation is

$$I_p \frac{d^2\theta}{dt^2} + Mgd\theta = 0 \ .$$

It is an equation of SHM with period *T* given by the relation

$$\frac{2\pi}{T} = \sqrt{\frac{Mgd}{I_p}} \ ,$$

or

$$T = 2\pi\sqrt{\frac{I_p}{Mgd}} = 2\pi\sqrt{\frac{\left(\frac{1}{2}R^2 + d^2\right)}{gd}} \ .$$

Substituting *R* =14.4 cm and *d* = 10.2 cm, we calculate the value of *T*,

$$T = 2\pi\sqrt{\frac{\left(\frac{1}{2}\times 14.4^2 + 10.2^2\right)\times 10^{-4}}{9.8\times 10.2\times 10^{-2}}} \text{ s,}$$

$$= 906 \text{ ms.}$$