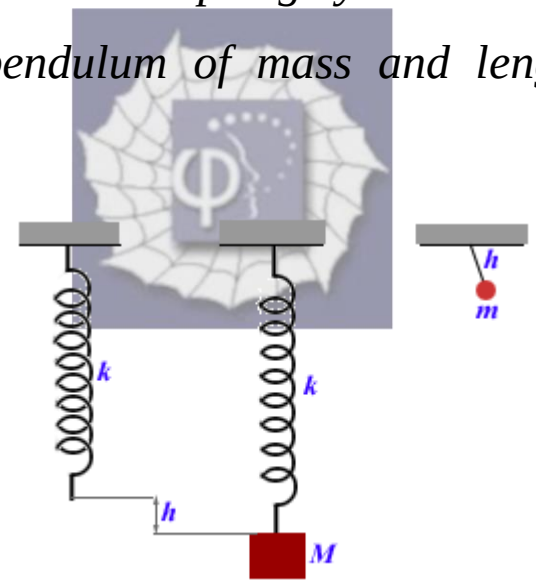
112.

Problem 15.42 (RHK)

There is an interesting relationship between the block-spring system and the simple pendulum. Suppose that we hang an object of mass M on the end of a spring, and when the object is in equilibrium the spring is stretched a distance h. We have to show that the frequency of this block-spring system is the sane as that of a simple pendulum of mass and length h, even if $m \neq M$.



Solution:

When a block of mass *M* is hung from the spring its length stretches by distance *h* determined by its force constant *k*,

$$M g = k h .$$

When the mass M is pulled down by a distance x from its equilibrium position, it is pulled up by the spring with force $k\,x$ and its equation of motion is

$$M\frac{d^2x}{dt^2}+kx=0\ .$$

It is a SHM with angular frequency ω

$$\omega=\sqrt{\frac{k}{M}}=\sqrt{\frac{g}{h}}\ .$$

Therefore the SHM of the mass M is equivalent to SHM of a simple pendulum of length h with a bob of mass m. As the SHM of the simple pendulum does not depend on the mass of the bob, above result holds even if $M\neq m$.