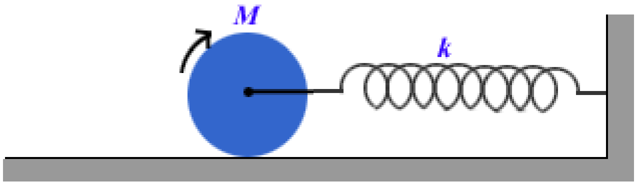
111.

<u>Problem 15.37 (RHK)</u>

A solid cylinder is attached to a horizontal massless spring so that it can roll without slipping along a horizontal surface (*see diagram*). *The force constant k of the spring is* 2.94 N/cm. *If the system is released from rest at a position in which the spring is stretched by* 23.9 cm, *we have to find* (a) *the translational kinetic energy and* (b) *the rotational kinetic energy of the cylinder as it passes through the equilibrium position*. (c) *We have to show that under these conditions the centre of mass of the cylinder executes simple harmonic motion with a period*

$$T = 2\pi\sqrt{3M/2k}\ ,$$

where M is the mass of the cylinder.



Solution:

(a) and (b)

A solid cylinder is attached to a horizontal massless spring so that it can '*roll without slipping*'.

Force constant k of the spring is

$$k = 2.94 \times 10^{-2} \text{ N m}^{-1} .$$

If the system is released from rest at a position when the spring is stretched by a = 23.9 cm, the total mechanical energy of the system is equal to the stored potential energy of the spring in that position, $\frac{1}{2}ka^2$. If we assume that the radius of the cylinder is R, its rotational inertia about the axis perpendicular to the plane of the diagram will be

$\frac{1}{2}MR^2$.

At the equilibrium position the spring will be relaxed and so its stored potential energy will be zero. Therefore, when the cylinder passes through the equilibrium position of its SHM, its mechanical energy will comprise of the translational kinetic energy and the rotational kinetic energy. Let v be the translational speed and ω be the angular speed of the cylinder when it passes through the equilibrium position of the spring. As the cylinder is rolling without slipping, ω and v are related as

$$v = \omega R.$$

Energy conservation equation is

$$\frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2 = \frac{1}{2}ka^2.$$

Substituting expressions for I and ω, and after making algebraic simplifications, we get

$$\frac{3}{2}Mv^2 = ka^2 \ .$$

Therefore, the translational kinetic energy of the cylinder at the equilibrium position of the spring is

$$\begin{aligned}
\frac{1}{2}Mv^2 &= \frac{1}{3}ka^2 \\
&= \frac{1}{3}\times 2.94\times 10^2 \times 0.239^2 \text{ J} \\
&= 5.59 \text{ J.}
\end{aligned}$$

And, the rotational energy of the cylinder

$$\frac{1}{2}I\omega^2 = \frac{1}{4}Mv^2 = 2.8 \text{ J} \ .$$

(c)

At a general displacement x of the spring from its equilibrium position the equation of energy conservation is

$$\frac{1}{2}\left(\frac{1}{2}MR^2\right)\times\frac{1}{R^2}\left(\frac{dx}{dt}\right)^2+\frac{1}{2}M\left(\frac{dx}{dt}\right)^2+\frac{1}{2}k\,x^2=\frac{1}{2}k\,a^2$$

or

$$\frac{3}{2}M\left(\frac{dx}{dt}\right)^2+k\,x^2=k\,a^2.$$

Differentiating this equation with respect to time, t, we get

$$3M\,\frac{dx}{dt}\times\frac{d^2x}{dt^2}+2k\,x\frac{dx}{dt}=0\ ,$$

or

$$\frac{d^2x}{dt^2}+\frac{2k}{3M}x=0\ .$$

This is an equation of SHM, with period T given by

$$\frac{2\pi}{T}=\sqrt{\frac{2k}{3M}},\quad \text{or,}\quad T=2\pi\sqrt{\frac{3M}{2k}}\ .$$