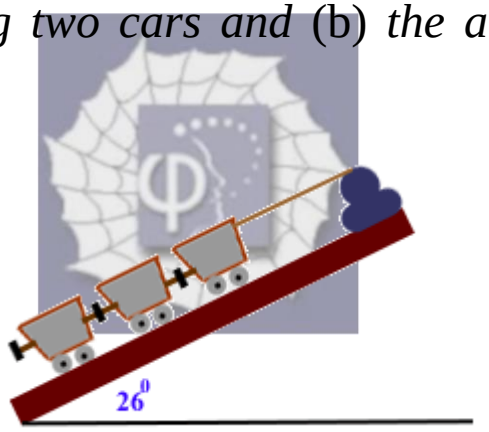
108.

Problem 15.23 (RHK)

Three 10,000 kg *ore-cars are held at rest on a* 26.0^0 *incline on a mine railway using a cable that is parallel to the incline. The cable is observed to stretch* 14.2 cm *just before coupling breaks, detaching one of the cars. We have to find* (a) *the frequency of the resulting oscillations of the remaining two cars and* (b) *the amplitude of the oscillations.*



Solution:

Mass of each car is 10,000 kg. Cable is stretched by 14.2 cm just before the cable breaks. We can determine the force constant of the cable using this information.

$$k \times 14.2 \times 10^{-2}\ \text{m} = 3 \times 10^4 \times 9.8 \times \sin 26^0\ \text{N},$$

or

$$k = 0.907 \times 10^6\ \text{N m}^{-1}.$$

Equation of motion of the remaining two cars after one of the cars gets detached from the cable is

$$2\times10^4\frac{d^2x}{dt^2}=-kx+2\times10^4\times9.8\times\sin26^0\ .$$

Let us call

$$2\times10^4\times9.8\times\sin26^0=k\,x_0\ ,$$

giving

$$x_0=\frac{2\times10^4\times9.8\times\sin26^0}{0.907\times10^6}=9.466\times10^{-2}\ \text{m.}$$

Equation of motion can be expressed as

$$\frac{d^2\left(x-x_0\right)}{dt^2}+\frac{0.907\times10^6}{2\times10^4}\left(x-x_0\right)=0\ .$$

It is an equation of SHM. As the SHM of the remaining two cars will be centred around $x_0=9.46$ cm, and the third car gets detached from the cable when it is stretched by 14.2 cm, the amplitude of oscillations will be

$$a=\left(14.2-9.46\right)\ \text{cm}=4.74\ \text{cm.}$$

Frequency of SHM will be

$$\nu=\frac{1}{2\pi}\sqrt{\frac{.907\times10^6}{2\times10^4}}\ \text{Hz}=1.07\ \text{Hz.}$$