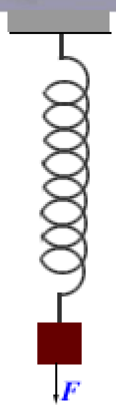
106.

<u>Problem 16.37P (HRW)</u>

A uniform spring whose unstretched length is L has a spring constant k. The spring is cut into two pieces of unstretched lengths L_1 *and* L_2*, with* $L_1 = nL_2$*. We have to find* (a) *the corresponding spring constants* k_1 *and* k_2 *in terms of n and k.* (b) *If a block is attached to the original spring, it oscillates with frequency f. If the spring is replaced with the piece* L_1 *or* L_2*, the corresponding frequency is* f_1 *or* f_2*. We have to find* f_1 *and* f_2 *in terms of* f*.*



<u>Solution:</u>

(a)

It is given that the unstretched length of the spring is L and that it has been cut into two pieces of lengths L_1 and

L_2, with $L_1 = nL_2$. Let the spring of force constant *k* stretch by length *x* under a force *F*. It may be appreciated that at equilibrium each section of the spring will experience the same force *F*. We will use this property to answer the problem.

Let us say that the portion of the spring with unstretched length L_1 stretches by length x_1 when the uncut spring stretches by length *x*. And, the portion of the spring with unstretched length L_2 stretch by length x_2.

We have

$$L = L_1 + L_2 = (n+1)L_2 \ ,$$

and

$$x_1 = \frac{n}{n+1}x \ , x_2 = \frac{1}{n+1}x \ .$$

As each portion of the spring is subject to the same force F, we can now find spring constant, k_1, for the portion with the length L_1, and the spring constant, k_2, for the portion with the length L_2, by the requirement

$$kx = k_1 \frac{n}{n+1}x, \text{ which gives } k_1 = \frac{(n+1)}{n}k,$$

and

$$kx = k_2 \frac{1}{n+1}x, \text{ which gives } k_2 = (n+1)k.$$

(b)

If the original spring oscillates with frequency f when a block of mass m is attached to it, then

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \ .$$

When the same block is attached to springs of lengths L_1 and L_2, there will be SHM with frequencies determined by k_1 and k_2, respectively. We have

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{(n+1)}{n}\times\frac{k}{m}} = \sqrt{\frac{(n+1)}{n}}\,f \ ,$$

and

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{(n+1)k}{m}} = \sqrt{(n+1)}\,f \ .$$