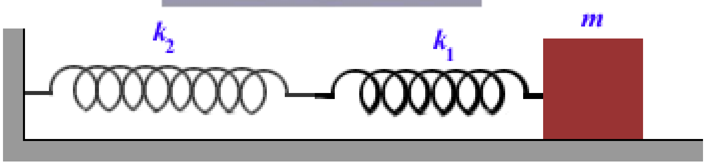
105.

<u>Problem 15.22(RHK)</u>

Two springs are joined and connected to a block of mass m (see diagram). The surfaces are frictionless. If the springs separately have force constants k_1 and k_2, *we have to show that the frequency of oscillation of the block is*

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{(k_1 + k_2)m}} = \frac{\nu_1 \nu_2}{\sqrt{\nu_1^{\,2} + \nu_2^{\,2}}},$$

where ν_1 *and* ν_2 *are the frequencies at which the blocks would oscillate if connected only to spring* 1 *or spring* 2.



<u>Solution</u>

Let the block be displaced to the right from its equilibrium position by distance *x*. Let the spring with force constant k_1 be stretched from its relaxed length by x_1 and the spring with force constant k_2 be stretched from its relaxed length by x_2. Then

$$x = x_1 + x_2 .$$

Let the restoring force on the block be F. As both the springs are stretched by force F, we have

$$F = k_1 x_1 \ ,$$

and

$$F = k_2 x_2 \ .$$

We thus have the relation

$$\frac{x}{F} = \left(\frac{1}{k_1} + \frac{1}{k_2} \right).$$

Let us define

$$\left(\frac{1}{k_1} + \frac{1}{k_2} \right) = \frac{1}{k} \ .$$

In terms of k, $F = kx$.

Equation of motion of the block of mass m is

$$m \frac{d^2 x}{dt^2} = -F \ ,$$

or

$$\frac{d^2 x}{dt^2} + kx = 0 \ .$$

It is an equation of SHM with frequency

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \ ,$$

where

$$k = \frac{k_1 k_2}{k_1 + k_2}.$$

Force constants k_1 and k_2, and mass m of the block correspond to two SHM with frequencies ν_1 and ν_2 given by the relations

$$\frac{k_1}{m} = (2\pi\nu_1)^2 \text{ and } \frac{k_2}{m} = (2\pi\nu_2)^2 \ ,$$

we get,

$$\nu = \frac{\nu_1 \nu_2}{\sqrt{\left(\nu_1{}^2 + \nu_2{}^2\right)}}.$$