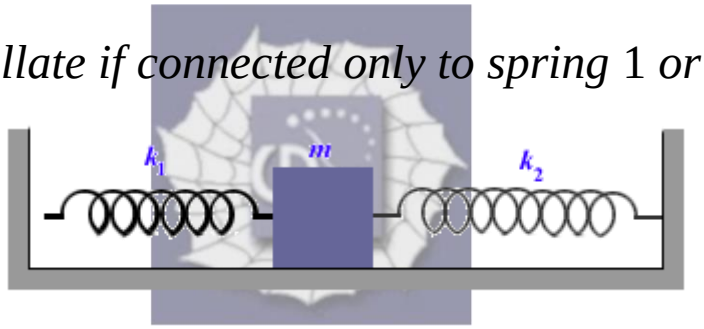
104.

Problem 15.21(RHK)

Two springs are attached to a block of mass m, free to slide on a frictionless horizontal surface. We have to show that the frequency of oscillation of the block is

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}} == \sqrt{\nu_1^{\,2} + \nu_2^{\,2}}\ ,$$

where ν_1 *and* ν_2 *are the frequencies at which the block would oscillate if connected only to spring* 1 *or spring* 2.



Solution:

Let the block of mass, *m*, at some instant, be displaced to from its equilibrium position to the right by distance *x*. Spring with spring-constant k_1 in this situation will be stretched and will therefore exert restoring force $k_1 x$ on the block in the direction right-to-left, and the spring with spring-constant k_2 will be compressed and will exert force $k_2 x$ on the block, also, in the direction right-to-left in order to bring the block to its undisturbed position.

Therefore, the force due to the combined action of the two springs will be

$-(k_1 + k_2)x$.

Equation of motion of the block is

$$m\frac{d^2x}{dt^2} + (k_1 + k_2)x = 0 \ .$$

It is an equation of SHM. Frequency of this SHM is

$$\nu = \frac{1}{2\pi}\sqrt{\left(\frac{k_1 + k_2}{m}\right)} = \sqrt{\nu_1^{\,2} + \nu_2^{\,2}} \ ,$$

as

$$\nu_1^{\,2} = \frac{1}{(2\pi)^2} \times \frac{k_1}{m}, \text{ and } \nu_2^{\,2} = \frac{1}{(2\pi)^2} \times \frac{k_2}{m} \ .$$