103.

Problem 16.27P (HRW)

A block is on a piston that is moving vertically with simple harmonic motion. Period of SHM is 1.0 s. *We have to find* (a) *the amplitude of motion that will separate the block and the piston.* (b) *If the piston has amplitude of* 5.0 cm, *we have to estimate the maximum frequency for which the block and piston will be in contact continuously.*

Solution:

(a)

Piston and block will separate when the downward acceleration of the piston exceeds *g*.

SHM period of the piston is 1.0 s. Angular frequency of SHM, ω, will therefore be

$$\omega = \frac{2\pi}{T} = 2\pi \text{ rad s}^{-1}.$$

Amplitude of acceleration, *x*, and the maximum acceleration, $a_{\max}$, during the SHM are related as

$$a_{\max} = \omega^2 x.$$

Amplitude of SHM for maximum acceleration to be g will therefore be

$$x_g = \frac{g}{\omega^2} = \frac{9.8}{4\pi^2}\ \text{m} = 25\ \text{cm.}$$

(b)

We find next the maximum frequency for which the block and the piston will be in contact continuously. We use the relation for SHM between the maximum acceleration and amplitude for a given frequency,

$$4\pi^2\nu^2 x = a_{\max}.$$

The piston and the block will be in contact continuously for $a_{\max} = g$. For $x = 5.0\times10^{-2}$ m, and $a_{\max} = g$, ν is

$$\nu = \left(\frac{9.8}{4\pi^2 \times 5.0\times10^{-2}}\right)^{1/2}\ \text{Hz},$$

$$= 2.2\ \text{Hz}.$$