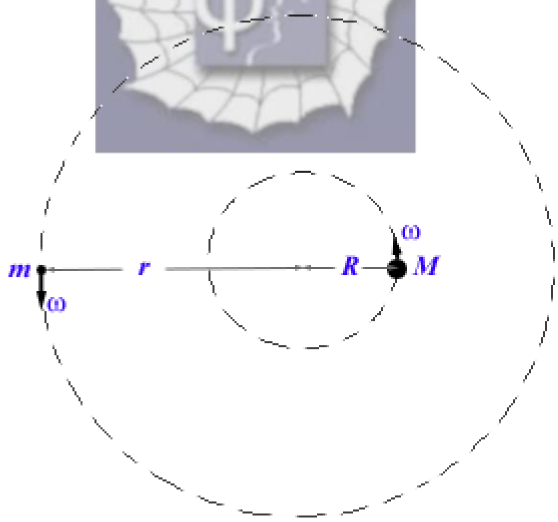
101.

<u>Problem 16.76 (RHK)</u>

We have to show that by use of the reduced mass concept the two body problem in gravitational field can be simplified to a one-body problem. That is, we have to show that if we use $\mu = mM/(m+M)$*, where* μ *is the reduced mass, we may solve for motion of m relative to M exactly as though M were the origin of our initial reference frame.*



<u>Solution:</u>

Let us consider the two-body problem when two particles are moving in circular orbits under the gravitational force exerted by each on the other.

From conservation of momentum and symmetry we may note that the angular speed of particle of mass *m* and that of the particle of mass *M* in their circular orbits will be equal and let us denote the angular speed by ω. Both particles will move in circular orbits with a common centre at the centre of mass of *m* and *M*. If r and R are the radii of their circular orbits, we have

$$mr = MR.$$

Using the universal law of gravitation and the expression for the centripetal force for circular motion, we have the following two equations of motion:

$$m\omega^2 r = \frac{GMm}{(r+R)^2} \; ,$$

and

$$M\omega^2 R = \frac{GMm}{(r+R)^2} \; .$$

These two equations are equivalent to the following two equations:

$$mr = MR,$$

and

$$\omega^2 (r+R) = \frac{G(m+M)}{(r+R)^2} \; .$$

We rewrite this last equation as

$$\omega^2 \frac{mM}{(m+M)}(r+R) = \frac{GmM}{(r+R)^2} \ ,$$

or

$$\omega^2 \mu (r+R) = \frac{GmM}{(r+R)^2} \ .$$

That is the problem reduces to a one-body problem with a particle of mass $\mu = mM/(m+M)$, reduced mass, moving as though M were at the origin of our initial frame and distance.

Also, the total energy of the system E is

$$E = \frac{1}{2} m (r\omega)^2 + \frac{1}{2} M (R\omega)^2 - \frac{GMm}{(r+R)} \ ,$$

$$= \frac{1}{2} \mu \left((r+R)\omega \right)^2 - \frac{GMm}{(r+R)} \ .$$