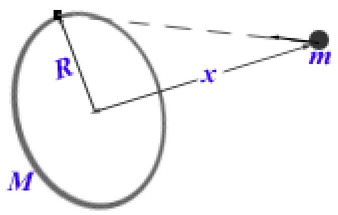
97.

<u>Problem 14.52P (HRW)</u>

Several planets (Jupiter, Saturn, Uranus) possess nearly circular surrounding rings, perhaps composed of material that failed to form a satellite. In addition, many galaxies contain ring like structures. Consider a homogeneous ring of mass M and radius R. We have to find (a) *the gravitational attraction exerted on a particle of mass m located at a distance x from the centre of the ring along its axis. Suppose the particle falls from rest as a result of the attraction of the ring of matter. We have to find* (b) *an expression for the speed with which it passes through the centre of the ring.*



<u>Solution:</u>

(a)

As the object of mass *m* lies on the axis of the ring, by symmetry the gravitational force exerted by the material in the ring will be along the axis. The magnitude of the gravitational pull experienced by the mass *m* is

$$F(x) = \frac{GMm}{R^2 + x^2} \times \frac{x}{\left(R^2 + x^2\right)^{1/2}}.$$

(b)

We next work out the gravitational potential energy of the system that comprises of the ring and the object of mass *m*.

$$V(x) = -\int_{\infty}^{x} \vec{F}(x).d\vec{x}.$$

As, $\vec{F}(x).d\vec{x} = -F(x)dx$, we have

$$V(x) = -\int_{\infty}^{x} \vec{F}(x).d\vec{x} = \int_{\infty}^{x} \frac{GMm\,x}{\left(R^2 + x^2\right)^{3/2}}.$$

Let us make the substitution

$$\left(R^2 + x^2\right)^{1/2} = \xi.$$

This implies

$$\frac{xdx}{\left(R^2 + x^2\right)^{1/2}} = d\xi.$$

We have

$$V\left(x\right) = GMm\int_{\infty}^{\xi}\frac{d\xi}{\xi^2} = -\frac{GMm}{\left(R^2+x^2\right)^{1/2}}.$$

In the problem it has been given that the particle of mass *m* falls from rest towards the ring. As *M*>>m, we can assume that the position of the ring remains unchanged during the motion of the particle. At the instant when the particle begins to be pulled by the ring its energy will be

$$E = -\frac{GMm}{\left(R^2+x^2\right)^{1/2}}.$$

When the particle passes through the centre of the ring $x = 0$. Let its speed at that instant be *v*. Then from conservation of energy we have

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{\left(R^2+x^2\right)^{1/2}}.$$

We thus find that when the particle passes through the centre of the ring expression for its speed is

$$v = \left\{2GM\left(\frac{1}{R} - \frac{1}{\left(R^2+x^2\right)^{1/2}}\right)\right\}^{\frac{1}{2}}.$$