92.

Problem 16.30 (RHK)

It is conjectured that a "burned-out" star could collapse to a "gravitational radius", defined as the radius for which the work needed to remove an object of mass m from the star's surface to infinity equals the rest energy mc^2 *of the object. We have to show that the gravitational radius of the Sun is* GM_S/c^2 *and determine its value in terms of the Sun's present radius.*

Solution:

Let us consider an object of mass *M* and surface radius *R*. Work required to remove an object of mass *m* from the surface of the object of mass *M* to infinity is the negative of the potential energy of the m-*M* system. It is

$$W_{escape} = \frac{GMm}{R}.$$

Gravitational radius of an object of mass *M* is the radius R_g which is such as would require energy mc^2 for an object of mass m to escape to infinity. That is

$$\frac{GMm}{R_g} = mc^2,$$

or

$$R_g = \frac{GM}{c^2}.$$

We calculate next the *gravitational radius* of the Sun.

$M_S = 1.99 \times 10^{30}$ kg,

Gravitationa constant $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}.

We use this data and find that the *gravitational radius* of the Sun is

$$R_g(Sun) = \frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{9 \times 10^{16}} \text{ m} = 1.47 \text{ km}.$$

According to the Einstein's theory of gravitation the correct value of R_g is $2GM/c^2$ and is called the Schwarzschild radius.