91.

Problem 16.19 (RHK)

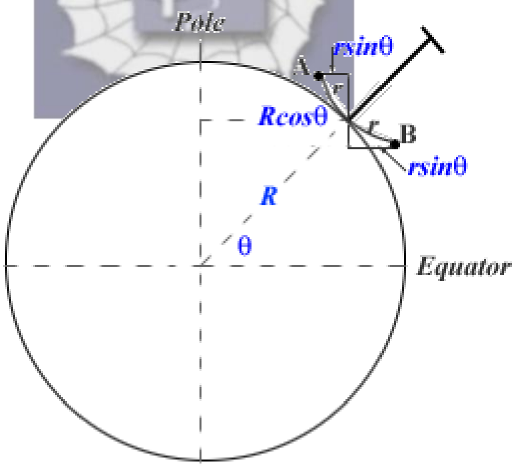
Foucault's Pendulum

A pendulum whose upper end is attached so as to allow pendulum to swing freely in any direction can be used to repeat an experiment first shown publicly by Foucault in 1851. *If the pendulum is set oscillating, the plane of oscillation slowly rotates with respect to a line drawn on the floor, even though the tension in the wire supporting the bob and the gravitational pull of the Earth lie in a plane.*

We have to show (a) *that this is a result of the fact that the Earth is not an inertial frame;* (b) *that for a Foucault pendulum at a latitude* θ*, the period of rotation of the plane, in hours, is* $24/\sin\theta$*;* (c) *behaviour of the pendulum in simple terms at the poles i.e. at* $\theta = 90^0$ and *at the equator i.e.* $\theta = 0^0$.

Solution:

To show the effect of Earth's West-East rotation Foucault considered a pendulum at a place at latitude θ. The bob of the pendulum is suspended from top in such a manner as to allow plane of oscillation of the pendulum to change freely. For analysis of the dynamics of oscillation let us assume that the amplitude of oscillation is r and that it remains constant over the period during which shift in the plane of oscillation of the pendulum is observed. To understand the dynamics an exaggerated diagram showing the oscillation of pendulum at latitude θ has been shown.



Foucault's Pendulum

As Earth is spinning around its polar axis in 24 hours, the angular frequency of its diurnal rotation ω is

$$\omega = \frac{2\pi}{24} \text{ rad hour}^{-1}.$$

Let R be the radius of the Earth. Speed of rotation about the polar axis of the mid-point of oscillation of the bob is $\omega R\cos\theta$. As shown in the diagram the end B of the swing of the pendulum is farther away from the North-South axis than the mid-point of oscillations by a distance $r\sin\theta$. The end B will move with speed of rotation about the North-South axis faster than the mid point of oscillation of the pendulum by an amount $\omega r\sin\theta$. Similarly, the speed of rotation of the end B of the swing of the pendulum will lag behind that of the mid-point of oscillation by an amount $\omega r\sin\theta$. Thus we may note that the end points B and A of the swing of the pendulum will move in a circle of radius r in anti-clockwise direction with speed $\omega r\sin\theta$.

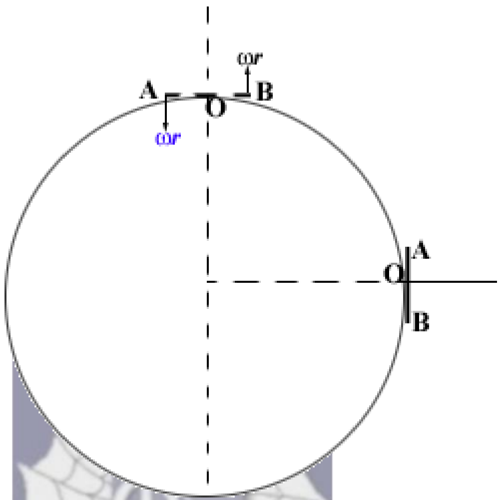
Time taken by the plane of oscillation of the pendulum to have turned by 360^0 will be

$$t = \frac{2\pi r}{\omega r\sin\theta} = \frac{24\text{ hour}}{\sin\theta}.$$

At the poles the circular speed of ends B and A will be ωr and therefore the plane of oscillation will turn full circle in 24 hours.

At the Equator both ends B and A will have the same angular speed of rotation about the North-South axis as the mid-point O. Therefore, at equator plane of oscillation of the pendulum will remain unchanged.

We may note that Foucault pendulum reveals that frame of reference tied up with the Earth is non-inertial and that the effect is more prominent at larger latitudes.



Foucault Pendulum at Pole and *at* Equator