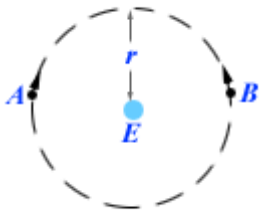
90.

Problem 16.50 (RHK)

Consider two satellites A and B of equal mass m, moving in the same circular orbit of radius r around the Earth E but in the opposite sense of revolution and therefore on a collision course. We have to find (a) *in terms of G,* M_E*, m and r the total mechanical energy of the two-satellite-plus-Earth system before collision.* (b) *If the collision is completely inelastic so that wreckage remains as one piece of tangled material, we have to find the total mechanical energy immediately after collision.* (c) *We have to describe the subsequent motion of the wreckage.*



Solution:

(a)

The total mechanical energy of the two-satellite-plus-Earth system will be the sum of the kinetic energy of the satellites *A* and *B*, gravitational potential energy of satellite-Earth system for each of the two satellites and the gravitational potential energy of the two satellites.

We neglect the contribution of potential energy of the two-satellite system in comparison to the potential energy of the Earth-satellite system.

In this approximation the potential energy of the system is

$$PE = -2\frac{GM_E m}{r}.$$

The kinetic energy of the two satellites is

$$KE = 2\left(\frac{1}{2}mv^2\right).$$

Therefore, the mechanical energy of the two-satellite-plus-Earth system is

$$E = mv^2 - \frac{2GM_E m}{r}.$$

As each satellite is in a circular orbit of radius *r*, the centripetal force is provided by the gravitational pull of the Earth on the satellite, that is

$$\frac{v^2}{r} = \frac{GM_E}{r^2}.$$

Therefore, the total mechanical energy of the system before the collision of the satellites with each other is

$$E = -\frac{GM_E m}{r}.$$

(b)

Just before the collision each satellite will be approaching each other with equal and opposite velocities. Therefore, the total momentum will be zero. In an inelastic collision momentum is conserved. As the collision is completely inelastic and the wreckage remains one tangled piece of material, the total mechanical energy of the system just after the collision will be the sum of the potential energy of each satellite-Earth system. That is

$$E' = -\frac{2GM_E m}{r}.$$

The amount of mechanical energy lost in the collision will be

$$E - E' = \frac{GM_E m}{r}.$$

(c)

The subsequent motion of the wreckage will be a radial free-fall toward the centre of the Earth (*in the vertical direction above the Earth*).