89.

Problem 16.33 (RHK)

A rocket is accelerated to a speed of $v = 2\sqrt{gR_E}$ *near the Earth's surface and then coasts upward. We have to show* (a) *that it will escape from the Earth;* (b) *that very far from the Earth its speed is* $V = \sqrt{2gR_E}$.

Solution:

As the speed of the rocket near the Earth's surface is $v = 2\sqrt{gR_E}$, its kinetic energy is

$$KE = \tfrac{1}{2}mv^2 = 2mgR_E.$$

Its potential energy near the Earth's surface is

$$PE = -\frac{GM_E m}{R_E}.$$

As the acceleration due to gravity g by definition is

$$g = \frac{GM_E}{R_E{}^2},$$

$$PE = -gmR_E.$$

Its energy, which is the sum of PE and KE, is

$$E = gmR_E.$$

An object with $E = 0$ can escape the gravitational pull of the Earth. As for the rocket $E>0$, it will escape from the Earth.

Very far away from the Earth, the PE of the rocket will reduce to zero. Therefore, its velocity far away from the Earth will be determined by

$$\frac{1}{2}mv^2 = E = gmR_E,$$

and will be

$$v = \sqrt{2gR_E}.$$