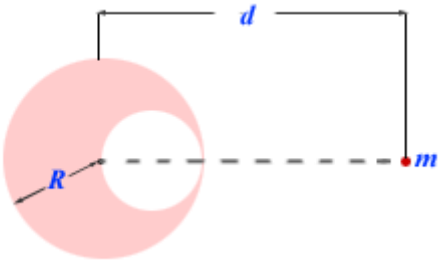
88.

Problem 16.23 (RHK)

A spherical hollow is made in a lead sphere of radius R, such that its surface touches the outside surface of the lead sphere and passes through its centre. The mass of the sphere before hollowing was M. We have to find the force, according to the law of universal gravitation, with which the hollowed lead sphere will attract a small sphere of mass m, which lies at a distance d from the centre of the lead sphere on the straight line connecting the centres of the spheres and of the hollow.



Solution:

We will solve this problem by applying the principle of superposition of gravitational forces.

Mass of the lead sphere, before the spherical cavity is carved out of it, as shown in the diagram, is *M*.

Therefore, its density ρ is

$$\rho = \frac{M}{\frac{4\pi}{3}R^3}.$$

Radius of the spherical cavity is $R/2$. Mass of the spherical lead sphere of radius $R/2$ will be

$$M' = \frac{4\pi}{3} \times \left(\frac{R}{2}\right)^3 \times \rho = \frac{M}{8}.$$

The gravitational force *F* on an object of mass *m* at a distance *d* from the hollowed sphere will, therefore, be equal to the force on *m* due to the solid lead sphere of mass *M* minus the force on it due to lead sphere of mass M' placed at the position of the spherical hollow. That is

$$F = \frac{GMm}{d^2} - \frac{GmM/8}{\left(d - R/2\right)^2},$$

$$= \frac{GMm}{d^2}\left(1 - \frac{1}{8\left(1 - \left(\frac{R}{2d}\right)^2\right)}\right).$$