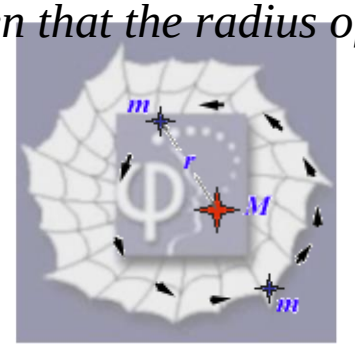
87.

Problem 16.61 (RHK)

A certain triple-star system consists of two stars, each of mass m, revolving about a central star, mass M, in the same circular orbit. The two stars stay at opposite ends of a diameter of the circular orbit. We have to derive expression for the period of revolution of the stars; we are given that the radius of the orbit is r.



Solution:

Force on any one of the stars of mass *m* due to the other star of mass *m* and the third star of mass *M* is

$$F = \frac{GMm}{r^2} + \frac{Gm^2}{r^2}.$$

As this star is revolving in a circular orbit of radius *r*, the gravitational force *F* on it has to be equal to the centripetal force for uniform circular motion, mv^2/r.

That is

$$\frac{mv^2}{r} = \frac{Gm}{r^2}\left(\frac{4M+m}{4}\right).$$

Substituting $v = 2\pi r/T$, where T is the orbital period of stars of mass m, we get

$$\frac{4\pi^2 r^2}{T^2} = \frac{G\left(4M+m\right)}{4r},$$

which gives

$$T = \frac{4\pi r^{3/2}}{\left(G\left(4M+m\right)\right)^{1/2}}.$$