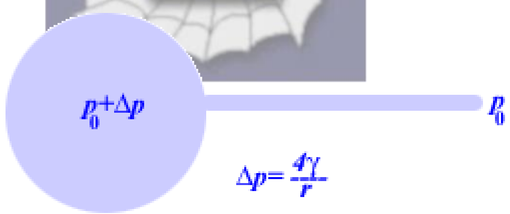
86.

<u>Problem 18.43 (RHK)</u>

A soap bubble of radius 38.2 mm *is blown on the end of a narrow tube of length* 11.2 cm *and internal diameter* 1.08 mm. *The other end of the tube is exposed to atmosphere. We have to find the time taken for the bubble radius to fall to* 21.6 mm. *We can assume Poiseuille flow in the tube. Surface tension of the soap bubble solution is* 2.50 N m^{-1}; *and the viscosity of air is* 1.80×10^{-5} N s m^{-2}.



<u>Solution:</u>

We will use two results one from surface tension and the other from viscosity for solving this problem.

i.

The gauge pressure Δp inside a soap bubble of radius *r* is

$$\Delta p = \frac{4\gamma}{r},$$

where γ is the surface tension of the soap solution.

ii.

According to Poiseuille’s law of viscous flow of fluid of density ρ in a pipe of radius *R*, length *L*, driven by pressur difference Δp is

$$\frac{dm}{dt} = \frac{\pi \rho R^4 \Delta p}{8 \eta L}.$$

As the air from inside the bubble is flowing out through the pipe, its radius will change with time. Let $r(t)$ be the radius of the soap bubble at time *t*. Mass of air inside the bubble at time t is

$$m(t) = \frac{4\pi r^3(t)\rho}{3},$$

where ρ is the density of air.

As the air flows out of the tube the rate of mass flux will be

$$\frac{dm(t)}{dt} = -4\pi\rho r^2 \frac{dr}{dt}.$$

Note, $\frac{dr}{dt} < 0$, as the radius of the bubble will decrease with time when air flows out of it.

With the above two results we can obtain a differential equation for $r(t)$,

$$-4\pi\rho r^2 \frac{dr}{dt} = \frac{\pi\rho R^r}{8\eta L} . \frac{4\gamma}{r} \ ,$$

or

$$r^3 dr = -\frac{R^4\gamma}{8\eta L} dt.$$

Integrating this equation, we find the variation of r as a function of time. It is

$$\frac{r^4(t)}{4} = -\frac{R^4\gamma}{8\eta L} t + c.$$

We solve for the constant of integration c by using the boundary condition that at $t = 0,\ r(0) = r_i$. This condition gives

$$c = \frac{r_i^4}{4}.$$

We thus obtain the relation

$$t = \frac{2\eta L}{R^4\gamma}\left(r_i^4 - r^4\right).$$

Data of the problem is

$$r_i = 38.2 \times 10^{-3} \text{ m},$$
$$r = 21.6 \times 10^{-3} \text{ m},$$
$$L = 11.2 \times 10^{-2} \text{ m},$$
$$R = 0.54 \times 10^{-3} \text{ m},$$
$$\gamma = 2.50 \times 10^{-2} \text{ N m}^{-1}, \text{ and}$$
$$\eta = 1.80 \times 10^{-5} \text{ N s m}^{-2}.$$

Substituting these values, we find that the time taken by the bubble radius to fall to 21.6 mm is 3617 s.