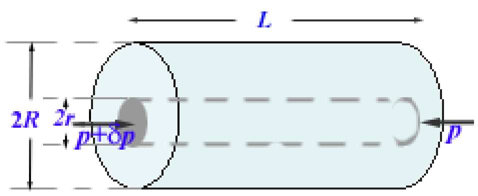
85.

<u>Problem 18.41 (RHK)</u>

A fluid of viscosity η *flows steadily through a horizontal cylindrical pipe of radius R and length L. By considering an arbitrary cylinder of fluid of radius r, we have to show* (a) *that the viscous force F due to the neighbouring layer is* $F = -\eta\left(2\pi rL\right)dv/dr$; (b) *that the force* F' *pushing that cylinder of fluid through the pipe is* $F' = \left(\pi r^2\right)\delta p$; (c) *by using the condition of equilibrium expression for* dv *in terms of* dr; *the relation*

$v = v_0\left(1 - \frac{r^2}{R^2}\right)$;(d) by finding an expression for the mass flux through the annular ring between r and $r + dr$, and by integrating it the total mass flux through the pipe.



<u>Solution:</u>

(a) and (b)

We consider a cylinder of fluid of radius *r* as shown in the diagram. As the fluid flow is laminar and steady, the net force on this element of fluid will be zero. The driving force for fluid motion is the pressure difference at the two ends of this cylinder. The force F' pushing the fluid in this cylinder pipe is

$$F' = (p + \delta p)\left(\pi r^2\right) - p\left(\pi r^2\right) = \delta p\left(\pi r^2\right).$$

From the definition of the coefficient of viscosity, the magnitude of the force of viscous drag on this cylindrical pipe of radius *r* exerted by the neighbouring layer at radius $r + \delta r$ is

$$F = \eta\left(2\pi r L\right)\left(-dv/dr\right).$$

As velocity of fluid decreases as *r* increases such that velocity is maximum at the axis of the cylindrical pipe and is zero at $r = R,\ \ dv/dr < 0.$

(c)

As the motion of the fluid in the cylindrical pipe of radius *r* is steady, the condition of equilibrium is

$$F' = F,$$

that is

$$-(2\pi r L)\eta \frac{dv}{dr} = (\pi r^2)\delta p,$$

or

$$dv = -\frac{\delta p\, r}{2L\eta} dr.$$

Integrating this equation, we get

$$v = -\frac{\delta p\, r^2}{4L\eta} + v_0 \ .$$

At the fluid in contact with the pipe does not move. This condition determines the constant of integration, v_0,

$$v_0 = \frac{\delta p\, R^2}{4L\eta}.$$

And

$$v = v_0\left(1 - \frac{r^2}{R^2}\right).$$

(d)

Amount of fluid flowing through the annular ring between radii r and $r + dr$ is

$$\frac{\Delta m}{\Delta t} = (2\pi r dr)\rho v(r)$$

$$= 2\pi\rho v_0 \left(1 - \frac{r^2}{R^2}\right) r dr.$$

The total mass flux through the cylindrical pipe is obtained by integrating the above expression from 0 to R.

$$\frac{dM}{dt} = 2\pi\rho v_0 \int_0^R r\left(1 - \frac{r^2}{R^2}\right) dr = \pi\rho v_0 \frac{R^2}{2}.$$

Substituting for v_0, we find

$$\frac{dM}{dt} = \frac{\pi\rho R^4 \delta p}{8\eta L}.$$