82. Problem 18.37 (RHK)

In the diagram a cross-section of the upper layers of the Earth has been shown. The surface of the Earth is broken into several rigid blocks, called plates that slide (slowly) over a slushy layer called asthenosphere. Suppose that the speed of the rigid plate shown is $v_0 = 48 \text{ mm y}^{-1}$, and that the base of the asthenosphere does not move. We have to calculate the sheer stress on the plate. The viscosity of the asthenosphere material is $4.0 \times 10^{19} \text{ Pa s}^{-1}$. We may ignore the curvature of the Earth.



Solution:

We will use the definition of viscosity of a fluid for solving this problem.

The ratio between stress and strain in a fluid is called the *coefficient of viscosity* η of the fluid. For fluids stress is the shearing stress and strain is the velocity gradient, $\frac{dv}{dy}$. That is

$$\eta = \frac{\text{shearing stres}}{\text{strain}} = \frac{F/A}{dv/dy} = \frac{F/A}{v/D}$$

Velocity gradient of the asthenosphere is

$$\frac{v}{D} = \frac{48 \times 10^{-3}}{3.156 \times 10^7 \times 190 \times 10^3} \text{ s}^{-1} = 8.0 \times 10^{-15} \text{ s}^{-1}.$$

Therefore, the stress on the base of the plate

$$=\eta \times \frac{v}{D} = 4.0 \times 10^{19} \times 8.0 \times 10^{-15}$$
 Pa=320 kPa