79.

Problem 17.55 (RHK)

We have to show that the pressure difference between the inside and the outside of a bubble of radius r is $4\gamma/r$*, where* γ *is the surface tension of the liquid from which the bubble is blown.*

Solution:

A soap bubble of radius *r*, comprises of two surfaces, each of area $4\pi r^2$; the inner and the outer surface of the bubble. Let γ be the surface tension of the solution. By definition γ is the surface energy per unit area. Therefore, the surface energy $U(r)$ of the bubble is

$$U(r) = 8\pi r^2.$$

Let the radius of the bubble be increased virtually by an infinitesimal amount Δr.

The source of additional surface energy

$$\Delta U(r) = U(r + \Delta r) - U(r) = 16\pi r^2 \gamma \Delta r$$

is the virtual work, $4\pi r^2 \Delta p$, done by the gauge pressure Δp inside the bubble. That is

$$\Delta p\, 4\pi r^2 \Delta r = 16\pi r \gamma\, \Delta r \ ,$$

or

$$\Delta \mathrm{p} = \frac{4\gamma}{r}.$$

As the bubble is in equilibrium the pressure inside the bubble is $p_0 + \frac{4\gamma}{r}$, where p_0 is the atmospheric pressure outside the bubble. Bubble experiences an inward force because of surface tension which is balanced by the gauge pressure inside the bubble.