76.

Problem 18.23 (RHK)

A jug contains 15 glasses of orange juice. When we open the tap at the bottom it takes 12.0 s *to full the first glass. We have to find the time taken when tap is kept open for the jug to empty out.*

Solution:

Let the cross-section of the jug be A m^2. Let height of juice at time t in the jug when the tap at the bottom of the jug is open be $h(t)$. Let the area of cross-section of the tap, which is at the bottom of the jug, be a m^2. Let the speed of flow of the juice at time t be $v(t)$.

From this data the initial height of the level of juice in the jug is H and the height when the first glass has been filled will be $^{14}\!/_{15}\,H$.

Also,

$$v(t) = \sqrt{2gh(t)}\,.$$

Therefore, rate of change of the level of juice in the jug will be

$$A\frac{dh(t)}{dt} = -av(t)\ ,$$

or

$$\frac{dh(t)}{dt} = -\frac{a}{A}\sqrt{2g}\left(h(t)\right)^{1/2}\ .$$

Integrating this equation, we find that

$$h^{1/2}(t) = H^{1/2} - \frac{a}{A}\sqrt{\frac{g}{2}}t\ .$$

We are given that the first glass is filled in 12 s. That is $h(12) = \frac{14}{15}H$. This gives

$$H^{1/2}\left(1-\left(\tfrac{14}{15}\right)^{1/2}\right) = 12\frac{a}{A}\sqrt{g/2}\ ,$$

or

$$H^{1/2}\frac{A}{a}\sqrt{\frac{2}{g}} = 354.$$

Time T for complete draining out of juice from the jug is given by $h(T) = 0,$ that is

$$T = H^{1/2}\frac{A}{a}\sqrt{\frac{2}{g}} = 354 \text{ s}.$$

We thus find that the time required for filling the remaining 14 glasses after the first glass has been filled will be 342 s or 5 min 42 s.