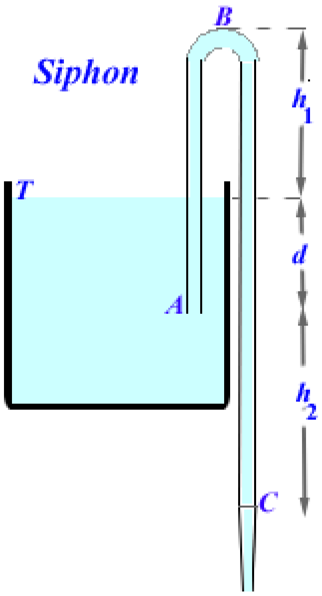
74.

Problem 18.21 (RHK)

A siphon is a device for removing liquid from a container that is not to be tipped. The tube must initially be filled, but once this has been done the liquid will flow until its level drops below the tube opening at A. The liquid has density ρ *and negligible viscosity. We have to find* (a) *the speed with which the liquid emerges from the tube at C,* (b) *the pressure in the liquid at the topmost point B, and* (c) *the greatest possible height* h_1 *to which a siphon may lift water.*



Solution:

In a siphon tube is initially filled by a process such as sucking the open end so that the liquid rises up in the tube and fills it. But once this has been done liquid will flow until the level in the tank drops to below the tube opening inside the liquid. However, there is an upper limit on the greatest possible height h_1 to which a siphon may lift liquid such as water.

(a)

We will investigate the siphon action by using Bernoulli's equation for hydrodynamics of streamlines,

$$p + \tfrac{1}{2}\rho v^2 + \rho g y = const.$$

We will measure vertical height y from the level of the opening of the end of the siphon tube inside the tank. We will fix the constant on a stream line by considering flow of liquid at the top level T in the tank. Let V be the speed of flow of liquid at T. Its height y is d. Let the atmospheric pressure at the top of the liquid level inside the tank and at the open end of the siphon be p_0. Right-hand-side of the Bernoulli's equation will, therefore, be

$$p_0 + \tfrac{1}{2}\rho V^2 + \rho g d = c.$$

As the cross-sectional area of the tank is much bigger than the cross-sectional area of the siphon tubes, V the speed of flow of the liquid at T will be small and so the term $\frac{1}{2}\rho V^2$ can be neglected in comparison to $p_0 + \rho g d$. In this approximation

$$c = p_0 + \rho g d.$$

We will next apply the Bernoulli's equation at points A, B, and C on a streamline that starts at the top level of the liquid in the tank and so the value of the constant is c.

At A the dynamic pressure p_A will not be the hydrostatic pressure $p_0 + \rho g d$ as the liquid is under motion. Let the speed of the liquid as it enters the siphon at A be v_A, then

$$p_A + \frac{1}{2}\rho v_A^2 = c.$$

At C, pressure on the liquid will be p_0, as at this end the siphon is open. The vertical height of the siphon at C measured from A is $-h_2$, so we have

$$p_0 + \frac{1}{2}\rho v_C^2 - \rho g h_2 = c.$$

Substituting $c = p_0 + \rho g d$, we get

$$v_C^2 = 2g\left(h_2 + d\right).$$

(b)

At *B*, let the pressure be p_B and the liquid speed be v_B. We have assumed that the point *B* is on the same stream line which passes through *T*, *A* and *C*. We also assume that the tube of the siphon has uniform cross-section and as we are considering the flow of an incompressible fluid, $v_B = v_C$. We, therefore, have the relation

$$p_B + \rho g\left(h_1 + d\right) = p_0 - \rho g h_2 ,$$

or

$$p_B = p_0 - \rho g\left(h_1 + h_2 + d\right).$$

(c)

The maximum height, *h*, of siphon can be determined by requiring that the least value of p_B can be zero. That is

$$p_0 - \rho g\left(h + h_2 + d\right) = 0.$$

In this equation we can adjust lengths h_2 and *d* each to be zero. This gives

$$h_{\max} = \frac{p_0}{\rho g}.$$

We can estimate $h_{\max}$ for water by substituting

$$p_0 = 1.01 \times 10^5 \text{ Pa},$$

and

$$\rho_{water} = 1.0 \times 10^3 \text{ kg m}^{-3}.$$

We find that the maximum height of a siphon for water is 10.3 m.

As liquid flow in a siphon is assumed to be streamline after the siphon action commences the speed of flow of the liquid at the open end is as though the liquid flow is a gravity flow in a tank at a total depth $d + h_2$.

After liquid exits from the open end *C*, because of conservation of the rate of flow and gain in speed due to free-fall, it will form a jet.