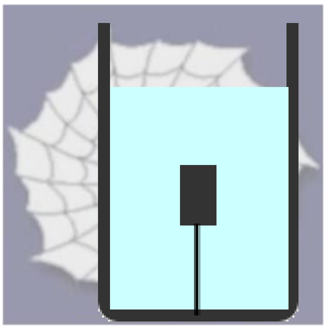
72.

Problem 17.28 (RHK)

The tension in a string holding a solid block below the surface of a liquid (of density greater than the solid) *is* T_0 *when the containing vessel is at rest. We have to show that the tension T, when the vessel has an upward acceleration a, is given by* $T_0\left(1+a/g\right)$.



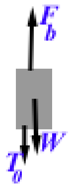
Solution:

Let the area of cross-section of the solid block be *A* and its height be *h.* Its volume *V* will be *Ah.* Let the density of the solid block be ρ_{solid}.

It is immersed in a liquid of density ρ_{liquid}. It is given that $\rho_{liquid} > \rho_{solid}$. The block is tied up to the container with a string. Free-body diagram of the block will be as shown below.

We will draw free-body diagram of the block in two situations: one when the container is at rest and two when the container is under upward acceleration *a*.

<u>Situation (i)</u>



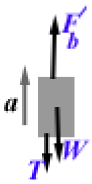
Let T_0 be the tension in the string. As the block is in equilibrium the net force in the vertical direction on it has to be zero. That is

$$T_0 = F_b - W.$$

W is the weight of the block and is equal to $\rho_{solid} V g$. F_b is the buoyant force on the block exerted by the liquid and is equal to the weight of the liquid displaced by the block. As the gauge pressure in a stationary liquid is $\rho_{liquid} g h$, $F_b = \rho_{liquid} V g$. We, therefore, have the relation

$$T_0 = Vg\left(\rho_{liquid} - \rho_{soild}\right).$$

<u>Situation (ii)</u>



In this situation the container, the liquid, and the block are under upward acceleration *a*.

As the liquid is under acceleration, the gauge pressure in it will vary with depth h as $\rho_{liquid} h(g+a)$.

Therefore, the buoyant force on the block in this situation will be

$$F_b' = \rho_{liquid} h(g+a).$$

As the string remains stretched when the block is under upward acceleration, tension T in it can be found by applying the Newton's second law of motion on the accelerated motion of the block. The equation of motion of the block in this situation is

$$\rho_{solid} Va = F_b' - W - T,$$

or

$$T = \rho_{liquid} V(g+a) - \rho_{solid} Vg - \rho_{solid} Va.$$

In the above expression by substituting

$T_0 = Vg\left(\rho_{liquid} - \rho_{soild}\right)$, we get

$$T = T_0 + \frac{a}{g} T_0 = T_0 \left(1 + \frac{a}{g}\right).$$