71.

Problem 17.26 (RHK)

Consider a container of fluid subject to a vertical upward *acceleration a.* (a) *We have to show that variation of gauge pressure with depth in the fluid is given by*

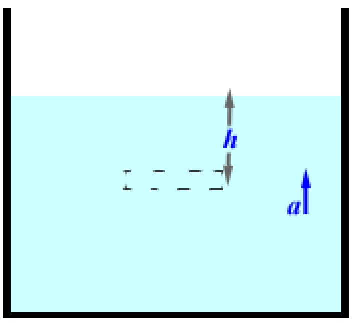
$$p = \rho h\left(g + a\right),$$

where h is the depth and ρ *is the density.* (b) *We have to show also that if the fluid as a whole undergoes a* vertical downward *acceleration a, the gauge pressure at depth h is given by*

$$p = \rho h\left(g - a\right).$$

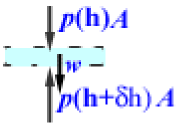
(c) *What is the state of affair in free fall?*

Solution:



We want to find variation of pressure with height in a fluid that as a whole is undergoing upward acceleration *a*. Density of the fluid is ρ.

Let us consider an element of the fluid at a depth *h* from the top of cross-sectional area *A* and width δh. Let the variation of hydrostatic pressure with depth be given by the function $p(h)$. The free-body diagram of the fluid element will be as shown below.



Downward forces acting on this fluid element will be its weight *w* and the force due to pressure at depth *h*, and the upward force on it will be due to the pressure at depth $h+\delta h$. As this fluid element is undergoing upward acceleration *a*, the equation of motion is

$$p(h+\delta h)A - p(h)A - \rho\delta hAg = \rho\delta hAa.$$

Retaining terms up to first order in δh, we get the following equation for the function $p(h)$,

$$\frac{dp}{dh} = (g+a)\rho.$$

Integrating this differential equation, we find that

$$p(h) = (g+a)\rho h + p_0 \; .$$

As p_0 is the pressure at the top layer of the fluid, the gauge pressure in the fluid varies as $(g+a)\rho h$.

(b) Carrying out a similar calculation for the situation when the fluid as a whole undergoes a vertical downward acceleration *a*, we will find that the gauge pressure at a depth *h* will vary as

$$p(h)=(g-a)\rho h.$$

(c) If the fluid is under free fall, that is $a=g$, the gauge pressure in the fluid will be zero.